Department of Engineering

Lecture 12: Power Flow

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Power Dissipated in Loads

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In this video we're going to talk about how much power gets dissipated in loads, especially loads that have some reactance.

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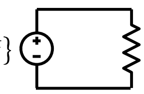
Average Power Dissipated in Load is Re{V*I}/2

$V_{zp}\cos(\omega t) = \mathrm{Re}\{V_{zp}e^{j\omega t}\}$  $Z = R + jX$

Real Valued Representation:

$$P(t) = V(t)I(t)$$

$$P(t) = V_{zp}\cos(\omega t)\,V_{zp}\cos(\omega t - \angle Z)/|Z|$$

$$P(t) = \frac{V_{zp}^2}{|Z|}\cos(\omega t)\cos(\omega t + \angle Z)$$

$$< P >= \frac{V_{zp}^2}{|Z|}\int_T \cos(\omega t)\cos(\omega t + \angle Z)\,dt$$

$$< P >= \frac{V_{zp}^2}{|Z|}\int_T (\cos(\omega t)\,(\cos(-\angle Z)\cos(\omega t) - \sin(-\angle Z)\sin(\omega t))dt$$

$$< P >= \frac{V_{zp}^2}{2|Z|}\cos(\angle Z)$$

Capacitors and inductors have angles of 90 → no power dissipated

Analytical Representation:

$$< P >= \frac{1}{2}\mathrm{Re}\{V^*(t)I(t)\}$$

$$< P >= \frac{1}{2}\mathrm{Re}\{V_{zp}e^{-j\omega t}V_{zp}e^{j\omega t}/|Z|e^{j\angle Z}\}$$

$$< P >= \frac{V_{zp}^2}{2|Z|}\mathrm{Re}\{e^{-j\angle Z}\}$$

$$< P >= \frac{V_{zp}^2}{2|Z|}\cos(\angle Z)$$

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I've drawn a very simple circuit here, just a supply driving a complex impedance, and we're going to find the power dissipated in the load in two ways to validate the analytical representation that I've referred to in earlier videos. Accordingly, I've represented the supply signal in two ways, as a real-valued quantity and as an analytic representation.

CLICK For our real-valued derivation, we start with the instantaneous power, which is the voltage across the load at time t multiplied by the current through the load at time t.
CLICK We can substitute in our real-valued supply voltage definition for V, and we can recognize that the impedance is a transfer function from voltage to current that will result in a phase shift and a scaling factor in the current value.
CLICK Pulling the constants out of the equation, we find that our instantaneous power varies as the product of two sinusoids with time. We also note that it has units of volts squared over some real number of ohms, which is encouraging.
CLICK However, we usually care about measures of average power more than we do instantaneous power, so we take an integral over one period of our sinusoids to find the average power.
CLICK That integral is simplified if we invoke the angle addition formula, which everyone definitely remembers from trigonometry.
CLICK The angle addition formula reveals that our second sinusoid can be expressed as a weighted sum of a sine and a cosine term, which is fortunate for our integral. The cosine

term will multiply with the other cosine to become cosine squared, and the integral of cosine squared over one period is one half.
CLICK The other term will be sine times cosine, and integrating that over one period has a value of zero.
CLICK That, and remembering that cosine is an even function, gives us a simple value for average power dissipated. This expression has some nice properties. Purely real loads simplify to V squared over 2R, which is consistent with results you've seen before. Further, purely imaginary results will result in the cosine term being equal to zero, indicating that no power is dissipated in ideal capacitors or inductors. One takeaway that's worth dwelling on is that power is dissipated when voltage and current are in phase. The cosine term here is showing how much the load moves current out of phase with the applied voltage.

Phew, that had a good result, but it was mathy and required me to invoke trigonometry black magic. I'm going to rederive the same result using our analytic representation to prove that we can get by with simpler computations, as long as we're willing to tolerate complex numbers.
CLICK We start with the expression I've asserted in the past, that the average power is equal to the complex conjugate of voltage multiplied by the current.
CLICK Then we substitute in our analytical representation of the supply voltage and note that we can find current by dividing the supply voltage by the load impedance. I've chose to represent the load impedance as a complex exponential here.
CLICK This simplifies nicely
CLICK and if we take the real part, we get the same result as before with less pain. We'll make a lot of use of this complex formulation of power, so make sure you're comfortable with it.

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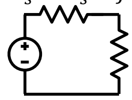
Max Power Transfer From Source if Zl=Zs*

$Z_s = R_s + jX_s$

$V_{zp}\cos(\omega t) = \mathrm{Re}\{V_{zp}e^{j\omega t}\}$  $Z_l = R_l + jX_l$

What's the average power in Zl?

$$< P_l >= \frac{1}{2}\mathrm{Re}\{V_l^*(t)I_l(t)\}$$

$$< P_l >= \frac{1}{2}\mathrm{Re}\left\{V_{zp}e^{-j\omega t}\frac{Z_l^*}{(Z_s+Z_l)^*}V_{zp}e^{j\omega t}\frac{1}{Z_s+Z_l}\right\}$$

$$< P_l >= \frac{V_{zp}^2}{2}\mathrm{Re}\left\{\frac{Z_l^*}{(Z_s+Z_l)^*}\frac{1}{(Z_s+Z_l)}\right\}$$

$$< P_l >= \frac{V_{zp}^2}{2}\frac{\mathrm{Re}\{Z_l^*\}}{|Z_s+Z_l|^2}$$

$$< P_l >= \frac{V_{zp}^2}{2}\frac{R_L}{|Z_s+Z_l|^2}$$

Maximizing average power in Zl

$$< P_l >= \frac{V_{zp}^2}{2}\frac{R_L}{(R_s+R_l)^2+(X_s+X_l)^2}$$

Control Zs?

Set Rs=0 and Xs=0, or as close as possible

Control Zl?

Set Xl=-Xs, and optimize Rl

$$\frac{d < P_l >}{dR_l} = \frac{V_{zp}^2}{2}\left(\frac{1}{(R_s+R_l)^2} - \frac{2R_L}{(R_s+R_l)^3}\right)$$

$$\frac{d < P_l >}{dR_l} = \frac{R_s - R_l}{(R_s+R_l)^3}$$ Set Rl=Rs

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One convenient application of this expression is proving a very useful theorem called the maximum power transfer theorem. This theorem tells us what values of Zs and Zl result in the most power being dissipated in Zl for the circuit pictured here.

CLICK We're going to start by finding the average power in Zl, then we're going to optimize it.
CLICK We have just proven the usefulness of analytic representations, so we're going to go ahead with an analytic representation of the load power in this problem.
CLICK We substitute in expressions for the load voltage and current in this step. The voltage on the load is set by the input driving the voltage divider between Zl and Zs, so we can see that divider ratio is conjugated in our expression here, and multipled by the conjugate of the input voltage here. We can also see that the current is set by the source voltage divided by the sum of Zl and Zs.
CLICK We factor and simplify a little bit, and we see that we have Zs+Zl multiplied by its complex conjugate in the denominator.
CLICK The product of a complex number with its conjugate is the magnitude squared, which we show in the denominator of this expression.
CLICK and finally, taking the real part of the load results in Rl in the numerator of this expression. This seems like a fine launching off point for optimization
CLICK so we copy the expression to the next column and expand out the magnitude in the

denominator to show both resistance and reactance of each impedance. This expression for load power has some obvious places to optimize, particularly in the denominator. Anything we can do to make the denominator smaller without affecting the numerator will get us closer to the maximum power.

CLICK If we control Zs then shrinking it will directly reduce the denominator without affecting the numerator. So making Rs and Xs as close to zero as possible helps.

CLICK If we control Zl then the first step is to cancel out the reactance of the source. The second is to optimize the value of Rl. Rl appears in both the numerator and denominator, which is why we need to treat it differently than Rs.

CLICK We differentiate the average power with respect to Rl here. We need to invoke the product rule, so we wind up with two terms.

CLICK Simplifying those terms, we find this expression goes to zero when Rl is equal to Rs. So we finish optimizing power by setting Rl equal to Rs. The overall load that we've designed here is referred to as a conjugate match: we want the same resistance as the source and the opposite reactance.

Summary

- Analytical representations of V and I simplify power calculations
- Power is dissipated in loads by voltage and current that are in phase
- Maximum power is transferred from source to load if the load impedance is a conjugate match of the source impedance
  - If you control the source, minimize Zs
  - If you control the load, conjugate match Zl to Zs

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Power Transfer Through Transmission Lines

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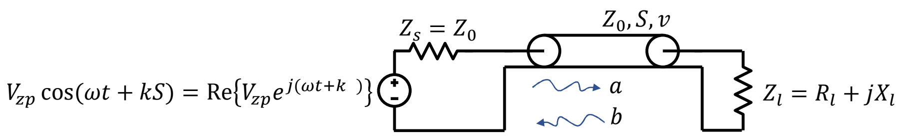
In this video we're going to increase the complexity of our modeling of power flow by adding transmission lines into our model. We'll also talk about representing power on a log scale.

Load Power Can Be Found From Wave Powers



Calculate load power from load voltage & current

$$< P_l >= \frac{1}{2}\mathrm{Re}\{V_l^*(t)I_l(t)\}$$

$$< P_l > = \frac{1}{2}\mathrm{Re}\left\{\frac{V_{zp}}{2}e^{-j\omega}\ (1+\Gamma)^*\frac{V_{zp}}{2Z_0}e^{j\omega t}(1-\Gamma)\right\}$$

$$< P_l > = \frac{V_{zp}^2}{8Z_0}\mathrm{Re}\{(1+|\Gamma|e^{-j\angle\Gamma})(1-|\Gamma|e^{j\angle\Gamma})\}$$

$$< P_l > = \frac{V_{zp}^2}{8Z_0}(1-|\Gamma|^2)$$

Calculate load power from wave conservation

$$< P_{wave} > = \frac{1}{2}|a|^2 = \frac{V_+^2}{2Z_0} = \frac{V_{zp}^2}{8Z_0}$$

$$\frac{1}{2}|a|^2 = \frac{1}{2}\mathrm{Re}\{V^*I\} + \frac{1}{2}|b|^2$$

$b = \Gamma a \rightarrow |b|^2 = |\Gamma|^2|a|^2$ ← Reflected power

$$\frac{1}{2}\mathrm{Re}\{V^*I\} = \frac{1}{2}|a|^2(1-|\Gamma|^2)$$

$$< P_l > = \frac{V_{zp}^2}{8Z_0}(1-|\Gamma|^2)$$

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I've drawn the circuit we'll be considering here, it's similar to the circuit we used to prove the Maximum Power Transfer Theorem, but we've added a transmission line between the source and the load and replaced Zs with a purely real impedance matched to the line. We'll take a look at what happens when Zs isn't matched in a later video, but for now it adds complexity without adding much understanding.

CLICK We're going to calculate power delivered to the load in two different ways to show that (1) they're the same and (2) the second way is pretty quick.
CLICK We know power in the load is going to be given by the complex conjugate of V times I.
CLICK And we can represent those values in terms Gamma and the wave amplitude, which is 1/2 of the generator amplitude because our source is matched. Recall that the voltage at the load is given by the sum of waves, so it's proportional to (1+Gamma), and the current is given by the difference of the waves, so its proportional to (1-Gamma). Also, note that we find the current of the forward going wave by dividing the amplitude of the voltage wave by Z0.
CLICK We can factor out Vzp and the factors of two to get a distinctive coefficient of Vzp squared over 8Z0. This coefficient is all over power flow calculations, so it's comforting when you find it. Notice that the factor of 8 comes from three factors of two: one from finding average power when driven by a sinusoid, and two from the amplitude of the

forward going wave being half of the generator voltage. We've also expressed Gamma in its complex exponential form, which has let us find the complex conjugate where needed.
CLICK The multiplication of these two Gamma terms leaves us with one minus the magnitude of Gamma^2. This is an intuitive result: high reflection coefficients should reduce the amount of power in the load. We're going to hone that intuition further by deriving this from conservation of power in the system.

CLICK Leaning on power conservation requires us to remember how much power is in a wave. Fortunately, we defined a to have an easy relation to power. Specifically power is one half of the magnitude of a squared. a was defined as the right travelling wave amplitude squared over two Z0, which is equal to Vzp squared over 8Z0 in this case.
CLICK Power conservation states that any power carried into the load by wave a needs to either be dissipated or reflected out on wave b
CLICK And we can express how much power is reflected in terms of reflection coefficient. We know b is Gamma times a because Gamma is defined as V- over V+, and if you multiply both sides of that equation by their complex conjugate you can find an expression for the magnitude of b squared. That value is the reflected power, and it's directly dependent on gamma.
CLICK Rearranging our conservation equation, we find that the power in the load has to be the incident power multiplied by 1 minus the magnitude of Gamma squared
CLICK and substituting for incident power, we find the same result as we had in the left column. However, by using the magnitude of Gamma squared as a kind of power gain, we were able to skip all the complex math. That's handy here, and it's really handy when working with measured quantities in lab, as we'll see on the next slide.

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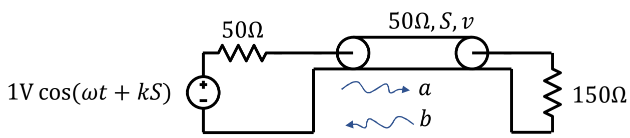
It’s Natural to Talk About Power Flow in dBm



Linear calculation of reflected power

$$< P_a > = \frac{1}{2}|a|^2 = \frac{1}{2}\left(\frac{1V}{2}\right)^2 \frac{1}{50} = 2.5\text{mW}$$

$$\Gamma = \frac{150-50}{150+50} = 0.5$$

$$< P_b > = \frac{1}{2}|a|^2|\Gamma|^2 = 2.5\text{mW} \cdot 0.25 = 0.625\text{mW}$$

$$< P_l > = \frac{1}{2}|a|^2(1-|\Gamma|^2) = 2.5\text{mW} \cdot 0.75 = 1.875\text{mW}$$

dB & power gain calculation of reflected power

$< P_a > = 4$ dBm 10*log(P/1mW)

$RL = 20\log(\Gamma) = -6\text{dB}$ Power ratio is Gamma^2

10^(-2/10)=0.63

$< P_b > = < P_a > + RL = -2$ dBm

Logs don’t add, so dissipation is tough to find

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In the spirit of this video set, we’re going to try calculating the power dissipated in the 150 ohm load in this example in two different ways.

CLICK we’ll try doing normal multiplying and adding first, and then we’re going to see if we can go even faster using logarithms.
CLICK The power in our incident wave is one half of the magnitude of a squared, which we find to be 1/4 of a milliWatt. As an aside, I find it handy to remember that the leading 1/2 and the 1/Z0 factor combine to a factor of 1/100 in 50 ohm systems.
CLICK Gamma is 0.5 for this example
CLICK So the reflected power is going to be one quarter of a milliwatt times 0.5 squared, which is 1 sixteenth of a milliwatt
CLICK The rest of the power goes into the load, so it receives 3 sixteenths of a milliwatt.

Fine, that seemed easy enough, but it can get kind of tough to carry power levels that are minute fractions of a milliwatt around in our heads. So we’re going to try this again with logarithms.
CLICK The power in this incident wave is -6 dBm. That’s a unit you might not have used a lot, but it means decibels relative to a milliwatt, and because decibels are defined by 10 times the log of a ratio of powers, you can think of it using the expression I’ve written on the right. I didn’t actually use that expression to calculate this value though: I just

remembered that -3dB corresponds to a factor of 1/2 in power, and we had two of those from our expression for wave power on the left.
CLICK Rather than finding the reflection coefficient for this approach, we're going to find a quantity called the return loss. This is the ratio of reflected power to incident power expressed in deciBels, and note that because power is proportional to Gamma squared, we're using 20 log in this expression rather than the 10 log we use for power quantities.
CLICK Logs are great because we can turn tricky multiplications into addition. We know the incident power is -6dBm, and we add our return loss to that to find that reflected power is -12dBm. That was super easy to do in our head! decibels and power gains remain popular in practice because you can add these log quantities in your head rather than having to multiply.
CLICK In case you can't do the conversion right off hand, 10 raised to the -12 over 10 is 0.063, so this answer gives us the same result as the left column.
CLICK However, logs have a downfall. We can't just find the difference between -6dBm and -12dBm to determine how much power is in the load because logs don't add that way. You have to convert your results back to linear quantities to use conservation relationships.

One final note, definitions of return loss are inconsistent, and it's sometimes defined as a positive quantity that you have to subtract from your incident power. This is fine, and it's easy to convert between definitions by remembering that your reflected signal should probably be smaller than your incident signal. Just make sure to read context clues when you're dealing with return loss

Summary

- Wave power conservation is an easy way to calculate load power.

$$\frac{1}{2}|a|^2 = \frac{1}{2}\mathrm{Re}\{V^*I\} + \frac{1}{2}|b|^2$$

- Return loss is the reflection coefficient expressed in dB

- Power gains in dB make it easy to find power levels in your head

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Power Gain

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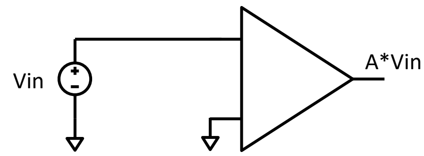
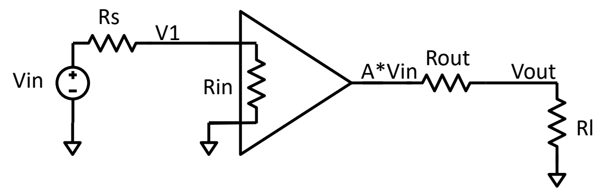
In this video we’re going to introduce a new measure of how effective an amplifier is at making a signal bigger called power gain, which is relevant because we care about how power flows through S parameter networks, especially amplifiers. However, we’re going to skip the S-parameters in this video so that we can get a sense of power flows without worrying about complex math. We’ll add S-parameters to our power gain analysis in a future video.

Power Gain Limits Performance w/ Small Load



Voltage gain is A



$$\frac{V_{out}}{V_{in}} = \frac{R_{in}}{R_s + R_{in}} A \frac{R_l}{R_{out} + R_l}$$

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One of the first things to discuss is the need for power gains at all. You've used amplifiers before, and we've mostly thought of a voltage source driving them, and the amplifier providing some voltage gain to that voltage source. For instance, this is probably how your learned to think about op-amps. Voltage gains seemed to work great up until now.

One argument against voltage gains is that we've been examining how convenient it is to do math with power. That math lets us avoid thinking about complex exponentials and it lets us compare signals of very different power levels using logs. That argument still holds water.
CLICK But this equation and amplifier model reveal another problem. Our first model presumes our voltage source is ideal or that the input impedance of the amplifier is really high, so that the amplifier doesn't load the voltage source. Neither of those are true at RF: we'll always be seeing 50 ohms of series impedance from matched lines, and it's hard to make very high input impedances because stray capacitance has low impedance when omega is big. We have a similar problem at the amplifier output, our loads are small and we might have some output impedance. These effects combine to make our loaded voltage gain, Vout/Vin, much smaller than the amplifier's voltage gain of A. So we'd like to make sure that our metric for gain takes impedances into account somehow.

The next question might be: "Why isn't loaded voltage gain a good enough metric? Why go to power gain?" As I mentioned above, power gain is convenient when you're working with signals at differing power levels, especially when they're multiplied by large, complex-valued gains. Further, voltages and voltage gains are often small at high frequencies, which makes them hard to measure. Power gains are easy to measure, and account for power carried in currents as well.

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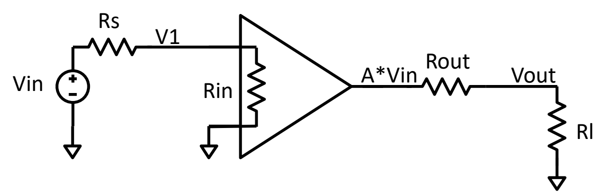
Some Powers are Easy to Measure



Useful power quantities:

$$P_s = V_{in}^2 \left(\frac{R_{in}}{R_s + R_{in}}\right)^2 \frac{1}{R_{in}}$$

$$P_{avs} = \frac{1}{2}\frac{V_{in}^2}{R_s + R_{in}}$$

$$P_l = A^2 V_{in}^2 \left(\frac{R_l}{R_{out} + R_l}\right)^2 \frac{1}{R_l}$$

$$P_{avl} = \frac{1}{2}\frac{A^2 V_{in}^2}{R_{out} + R_l}$$

Possible Power Gains:

$$G_P = \frac{P_l}{P_s}$$ Operating power gain, intuitive, easy to measure

$$G_T = \frac{P_l}{P_{avs}}$$ Transducer gain, honest about loading

$$G_A = \frac{P_{avl}}{P_{avs}}$$ Available power gain, best we could do

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With that said, we’re going to calculate some interesting values of power that might be used in calculating a power gain.

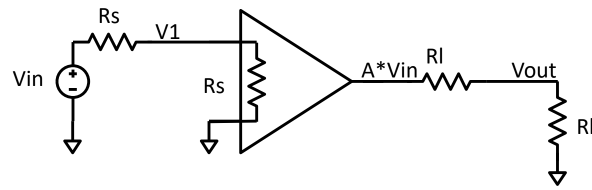
CLICK The first is the power from the source. This is the power that came out of the source and makes it into our amplifier. It’s given by the voltage on Rin squared over Rin. The voltage across Rin is set by a voltage divider attached to Vin, so we get a big squared ratio in this expression. Note that you could measure this value by measuring V1, even if you didn’t know Rin exactly.
CLICK We’ll also consider the power available from the source, which is the power that would come out of the source and into the load in the best-case scenario of a matched load. Vin squared over Rs plus Rin is the power coming out of the element Vin, and half of that makes it into the amplifier’s Rin in the best case.
CLICK We need to compare source power to power dissipated in the load, which is given by the voltage across the load squared divided by Rl. The load voltage is set by a divider between the amplifier output, A times Vin, and Vout.
CLICK Finally, the power available from the load is the best-case power delivered to the load when Rout is matched to Rl. As with the power available from the source, half of the power that comes out of the amplifier winds up in the load in this case.

CLICK We’re going to start making ratios of these quantities to define power gains. The first

one I've put up here is the most intuitive: it's the ratio of power in the load to power from the source. This is called the operating power gain, because it reflects what the amplifier does once power gets into it. This is easy to measure at low frequencies by grabbing the voltages at V1 and Vout. However, this measure is a little bit dishonest. The denominator is artificially small because it's neglecting power that is being dissipated in Rs.
CLICK We define a quantity called the transducer gain to fix that. This compares the power delivered to the load to the maximum power that could come from the source. It takes a penalty for loading at both the input and the output of the amplifier, so it's the most pessimistic of the power gains. That's useful in analysis because it reflects the actual performance of the amplifier.
CLICK Finally, we sometimes want to know the most optimistic gain for a system, which lets us figure out an upper bound on performance. That quantity is called the available power gain, and it's given by the power available to the load divided by the power available to the source.

Power Gains are All Equal in Matched Systems

Let $R_s = R_{in},\ R_{out} = R_l$



Useful power quantities:

$$P_s = V_{in}^2\left(\frac{R_{in}}{R_s + R_{in}}\right)^2 \frac{1}{R_{in}} = \frac{V_{in}^2}{4R_s}$$

$$P_{avs} = \frac{1}{2}\frac{V_{in}^2}{R_s + R_{in}} = \frac{V_{in}^2}{4R_s}$$

$$P_l = A^2 V_{in}^2\left(\frac{R_l}{R_{out} + R_l}\right)^2 \frac{1}{R_l} = \frac{V_{in}^2}{4R_l}$$

$$P_{avl} = \frac{1}{2}\frac{A^2 V_{in}^2}{R_{out} + R_l} = \frac{V_{in}^2}{4R_l}$$

Possible Power Gains:

$$G_P = \frac{P_l}{P_s} = A^2\frac{R_s}{R_l}$$

$$G_T = \frac{P_l}{P_{avs}} = A^2\frac{R_s}{R_l}$$

$$G_A = \frac{P_{avl}}{P_{avs}} = A^2\frac{R_s}{R_l}$$

All of these gains are much easier to calculate if your system is matched at the input at the output. That's because the power from our source becomes the same as the available power from the source. Similarly, the power delivered to the load becomes the same as the power available to the load. As a result, all the power gains converge to the same value. That value is proportional to voltage gain squared, which makes sense because power is proportional to voltage squared.

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Summary

- Power gain is important because it accounts for impedances of sources and loads.

- There are many types of power gains
  - Pl/Ps – Operating power gain
  - Pl/Pavs – Transducer gain
  - Pavl/Pavs – Available power gain

- All power gains are the same if the system is matched.

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Power Flow, Power Gain and S-Parameters

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In this video we're going to derive expressions for power gain using two-port S-parameters.

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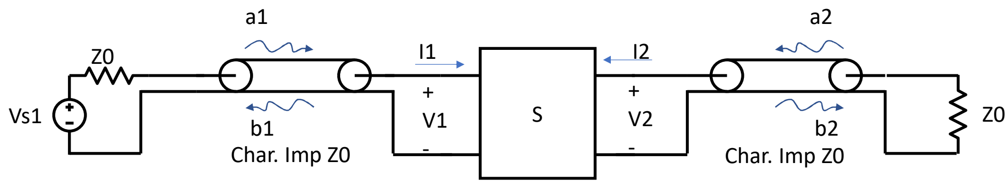
Power Gain Closely Related to S-Parameters



Useful power quantities:

$$P_s = \frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2 = \frac{1}{2}|a_1|^2(1 - |S_{11}|^2)$$

$$P_{avs} = \frac{1}{2}|a_1|^2$$

$$P_l = \frac{1}{2}|b_2|^2$$

$$P_{avl} = \frac{1}{2}|b_2|^2 - \frac{1}{2}|a_2|^2$$

Possible Power Gains:

$$G_P = \frac{P_l}{P_s} = \frac{|S_{21}|^2}{1 - |S_{11}|^2}$$ Pretend all power gets in

$$G_T = \frac{P_l}{P_{avs}} = |S_{21}|^2$$ Most common definition

$$G_A = \frac{P_{avl}}{P_{avs}} = \frac{|S_{21}|^2}{1 - |S_{22}|^2}$$ Pretend all power gets to load

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We're going to calculate the same useful power quantities we did in the last video, then calculate power gains with them, but we'll use S-parameters instead of real resistances this time. Note that our system has the source and the load impedances matched to the transmission lines. We'll relax that constraint on the next slide, but not matching the source and the load to transmission lines mostly has the effect of making our expressions confusing, and it's rare to see highly unmatched systems in the lab, so we're skipping it for now.

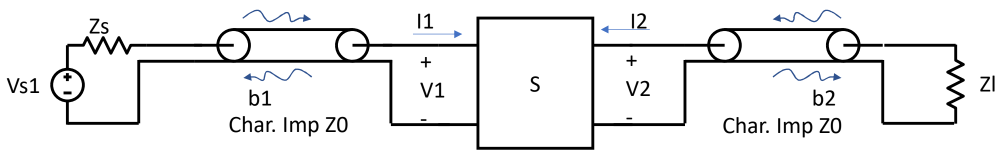
CLICK The power delivered from our source is the difference between the power in the a1 wave and the power in the b1 wave, and we can express that in terms of S11 because our load is matched to the two-port transmission line, which guarantees that a2 is zero and b1 is entirely determined by the reflection of a1.
CLICK The power available from the source presumes the S11 is zero, which is the same as saying port 1 of the S parameter network is matched to Z0, so all of the a1 wave goes into the S parameter network.
CLICK All of the power in the b2 wave is delivered to the load because the load is matched.
CLICK The power available to the load is given by the power in the b2 wave minus the power in the a2 wave.

The power gains can be calculated as ratios of these power quantities.

CLICK The operating power gain is given by the magnitude of S21 squared divided by one minus the magnitude of S11 squared. I made the claim in the last video that operating power gain was optimistic because it neglected power from the source that didn't make it into the amplifier. That's really explicit in the denominator of this expression: 1 minus the magnitude of S11 squared is a factor that corrects for reflected power.
CLICK The transducer power gain simplifies to the magnitude of S21 squared. This is the definition you'll see used most often.
CLICK The available power gain is given by the magnitude of S21 squared divided by one minus the magnitude of S22 squared. The denominator of this expression makes it very clear that we're presuming all of the power from the S network makes it into the load. …

Note that all of these expressions simplify to the magnitude of S21 squared if we're matched at the input and output of our S parameter network such that S11 and S22 are zero.

Unmatched Systems Have Complicated Gains



$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}$$

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0}$$

$$G_T = \frac{(1 - |\Gamma_s|^2)|S_{21}|^2(1 - |\Gamma_l|^2)}{|(1 - S_{11}\Gamma_s)(1 - S_{22}\Gamma_l) - \cancel{S_{12}S_{21}}\Gamma_s\Gamma_l|^2}$$

$$G_T = \frac{(1 - |\Gamma_s|^2)}{|(1 - S_{11}\Gamma_s)|^2}|S_{21}|^2\frac{(1 - |\Gamma_l|^2)}{|(1 - S_{22}\Gamma_l)|^2}$$

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We can still find power gains if we relax the constraint that the source and load impedances are matched. I've defined reflection coefficients off of the source and the load on this slide, and I've also given you a transducer power gain with no justification. The math is uglier, but you can see that this reduces to the magnitude of S21 squared if Gamma_s and Gamma_l are zero. Great!

CLICK Another interesting way to interpret this expression arises in networks where if S12 is zero. S12 being zero makes our system is unilateral, such that power only goes form port 1 to port 2. If that's the case, then we can rearrange the terms in the expression, into a quantity that depended only on S11 and Gamma_s, multiplied by the magnitude of S21 squared, multiplied by a quantity that depends only on S22 and Gamma_s. The S11 and S22 quantities are referred to as mismatch factors, and you can picture power trying to get into the S parameter network through the S11 and Gamma_s matching network, power getting scaled by S21, and then power trying to get out of the network through the S22 and Gamma_l matching network.

HARVEY MUDD COLLEGE Department of Engineering

Summary

- Power gains are easy to calculate from power conservation in a and b waves, and easy to express in S-parameters

$$G_P = \frac{P_l}{P_s} = \frac{|S_{21}|^2}{1 - |S_{11}|^2}$$

$$G_T = \frac{P_l}{P_{avs}} = |S_{21}|^2$$

$$G_A = \frac{P_{avl}}{P_{avs}} = \frac{|S_{21}|^2}{1 - |S_{22}|^2}$$