Department of Engineering

Feedback in Amplifiers

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E151 – Analog Circuit Design

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In this video series we're going to start analyzing how feedback affects amplifiers. We've spent a long time building up to making an op-amp, but op-amps are designed to be used in feedback, which has big implications on their performance. So we're taking time in this video series to step back to control theory and apply it to common circuit situations. We'll find that feedback is dangerous, it can cause instability, but also that it's very powerful, allowing us to modulate impedances, make gains that are independent of process, voltage and temperature variations, and reduce circuit non-idealities. We'll use feedback to make circuits work better in this video series, then explore how we can use it to make oscillators in the next video series.

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Why Feedback? (Also, Loop Gain and Static Error)

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In this video we're going to justify the use of feedback by listing its benefits. We're also going to define our basic vocabulary about feedback, which is surprisingly extensive.

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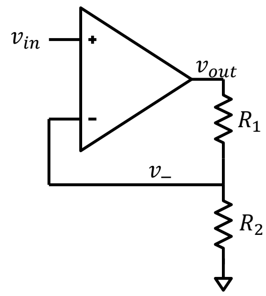
Systems in Feedback can Be Unstable

- Feedback comes with a risk of instability, why use it?

- Feedback can:
  - Control gain
  - Modulate input and output impedance
  - Reduce offsets
  - Improve linearity / reduce dead zones



$$\frac{v_{out}}{v_{in}} \approx 1 + \frac{R_2}{R_1}$$

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We're going to be talking about a lot of good things that can come out of feedback in this video, but it's crucial to acknowledge that feedback can cause instability. Any time you commit to designing feedback into a system, you also need to commit to a thorough stability analysis. An unstable system is usually one that doesn't meet its basic functional requirements, so failing to thoroughly account for instability, especially when users push the system in weird ways like high temperature operation or crazy loads, is a recipe for a lot of wasted effort and broken parts.

With that warning out of the way, there are a lot of reasons to use feedback. The most commonly cited of these is controlling gain reliably. The op-amp circuit on the right is configured for non-inverting gain, so its output voltage is related ot the input voltage by a gain of 1+R2/R1. That gain is independent of the circuit details of the op-amp, so it doesn't care that VBEON values wander around with temperature, or that our gain might depend on the exact value of ro. Even better, it's possibly to manufacture very precise resistors, and resistor ratios are temperature independent, so you can control your gain very effectively using a pair of precise passive components. We'll take a very close look at gain control on the next slide.

Feedback can also change the appearance of different impedances. We'll take a close look at that in the next video, but we've actually already put impedance modulation to use in some of our single stage amplifiers. Emitter followers have some feedback in them that results in the boosted input impedance. That feedback is also why emitter followers are susceptible to instability with some loads.

We're not going to go over the math on the final two bullets on the slide about offsets and linearity. However, there are quick qualitative arguments for each of them. If you have a small offset voltage on an amplifier in negative feedback, then the output is going to move to cancel out the offset because the output is forced to move to reduce error. The op-amp is constrained to drive the inputs together even if it has a dead zone at the output, so the output will just run past the boundaries of the dead zone in order to cancel a difference at the input. That's why class B amplifiers are often acceptable in op-amps even though they look like they should cause distortion.

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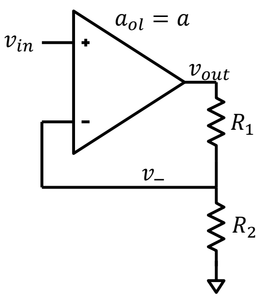
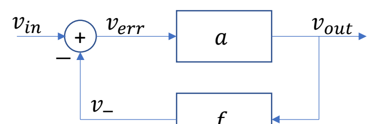
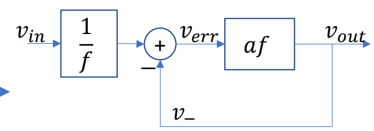
Gain Control Leads to ϵ_{ss}, Loop Gain is Useful



$$\frac{v_-}{v_{out}} = \frac{R_2}{R_1 + R_2} \stackrel{\text{def}}{=} f$$



$$a_{CL} = \frac{v_{out}}{v_{in}} = \frac{a}{1+af} = \frac{1}{f} \cdot \frac{af}{1+af} = \frac{1}{f}(1 - \epsilon_S)$$



$$\frac{v_{out}}{v_{in}} = \frac{1}{f}\frac{L}{1+L} = \frac{1}{f}\left(1 - \frac{1}{1+L}\right)$$

Name	Symbol	In This Case it is	How to Calculate
Open Loop Gain	a_{OL}	a	Circuit analysis or given.
Feedback Factor	f	$R_2/(R_1 + R_2)$	Circuit analysis or given.
Closed Loop Gain	a_{CL}	$a/(1 + af)$	Forward path / (1+loop gain)
Loop Gain	L	af	Product of blocks around loop
Commanded Gain	G	$1/f$	What's in front of unity feedback?
Static Error	ϵ_S	$1/af$	$1/(L + 1)$ for proportional feedback.

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So let's look at this first application of gain control. Doing so will give us an opportunity to define lots of useful feedback terms that give us a useful generic language to talk about feedback control.

We can get started by looking at the op-amp in feedback on the left. I've added a few details since the last slide. I've defined the gain of the op-amp, often called the open-loop gain, as a. I've also defined an additional variable called f, which is referred to as the feedback factor. In our analysis, the feedback factor is always going to be a constant that is smaller than one, but there are some fancy control techniques that let f become a function of s.

CLICK We can do a feedback analysis of this op-amp: the negative terminal is related to the output by a factor of f, and the difference between the input voltage and the negative terminal is multiplied by the differential mode gain of the op-amp, a. That gives rise to the block diagram pictured in the middle of the slide. Analyzing that block diagram tells us the relation between vout and vin. I always do that analysis using Black's Formula, which says the closed loop gain is equal to the forward parth over one plus the loop gain, or a over 1+af in our case. You can gain some extra insight into this gain by first factoring 1/f out of the numerator and then recognizing that af/(1+af) is very close to one. So that means the overall gain is going to be 1/f multiplied by some small error expression, and the deviation from 1 in that expression is called the static or steady-state error.

CLICK That was some simple algebra, but it relied on a whole lot of vocabulary words. So I've included this table so that we can round up all of those terms.

- The open-loop gain is the gain of the op-amp itself. Right now it's a constant a, but the open-loop gain often has dynamics inside of it too. More on that in a few videos.
- The feedback factor is f, which is R2/(R1+R2) for this circuit. For us, this will always be a constant that is smaller than 1, but you can play clever tricks by introducing dynamics or gain into it.
- The closed loop gain is vout/vin for the whole system. I don't like this term because people often call this just "gain", which can get confusing when you're trying to find circuit details. You calculate it using Black's formula, which says that it is the forward path / (1+loop gain).
- We need to know what the loop gain is in order to use that formula. It's the product of all the gains around the loop. Sometimes this is called the loop transmission and assigned the symbol T.
- I often introduce this auxiliary term called the commanded gain, which is the gain that our feedback factor suggests we should have. This is often the designer's intended gain before static error messes things up. I introduce this term to disambiguate this idea from closed loop gain. It's common for designers to use those terms interchangeably, which is confusing. We'll see a neat way to calculate the commanded gain in just a second.
- Finally, the static error is the difference between the commanded gain and the closed loop gain. In this case it's 1/af, which you could figure out by doing a bit of algebra above, but we'll see that it's always 1/(L+1) for our contant feedback factors in just a second.

CLICK One way that I like to analyze these feedback systems is by pushing the feedback factor through the summing junction, which gives us an informative block diagram. The 1/f block in front of the block diagram is our commanded gain, and the feedback loop consists of our loop gain in the forward path with unity gain feedback. That means the feedback look gain is L/(1+L), which is close to 1 as long as L is big. This is how I usually think of feedback loops, providing a commanded gain block, followed by a block that is trying its hardest to look like a factor of 1. That's the basis for the trick for calculating commaned gain in this table. One nice thing about this formulation is that you can rewrite L/(1+L) as (1-1/(1+L)), which makes it clear that your static error is always one over the loop gain +1. We'll see this loop gain +1 factor show up again, so this is a handy mnemonic.

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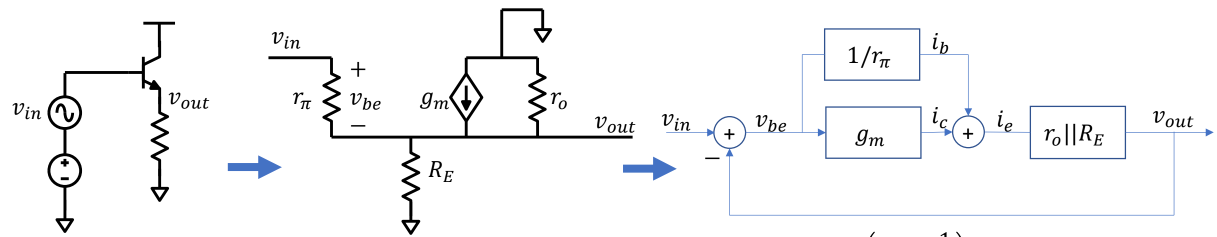
Don't Need Two Terminals for Feedback



$$a_v = \frac{(\beta+1)(R_E||r_o)}{r_\pi + (\beta+1)(R_E||r_o)}$$

$$v_{out} = (R_E||r_o)\left(i_{in} + g_m(i_{in} r_\pi)\right)$$

$$v_{in} = v_{out} + i_{in} r_\pi$$

$$\frac{v_{out}}{v_{in}} = \frac{\left(g_m + \frac{1}{r_\pi}\right)(r_o||R_E)}{1 + \left(g_m + \frac{1}{r_\pi}\right)(r_o||R_E)}$$

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We're going to be looking at feedback for lots of op-amp systems, and those systems always have two terminals. However, you don't need two terminals on a circuit to include feedback in it, and we can see that by observing that feedback is actually baked into the operation of an emitter follower. I included this slide for two reasons, first to show that one terminal circuits can have feedback in surprising places, and second to show that the emitter follower is really easy to model using feedback analysis, which I find very elegant.

I've included an emitter follower on the left of this slide and I've included our expression for emitter follower gain below it. That's just there as a helpful reminder.

CLICK It's easy to draw a small signal model for the emitter follower. We did a bunch of circuit analysis on this in order to find the gain expression on the left. As a reminder, that analysis started with finding the total current into the emitter resistor by observing that both the test current and gm generator current would flow in it. Then we rebuilt the input voltage from the voltage across the emitter resistor.

CLICK However, we could also express that calculation using a block diagram. Here the difference between vin and vout is labeled as vbe, and that vbe value drives a collector current through the gm generator and a base current through rpi. Those two currents add up to give us the total emitter current, which gets multiplied by the resistance hanging off the emitter to ground to calculate vout. This is exactly the same calculation we did with circuit analysis, just captured in block diagram form. I think this is clever, and when I find feedback in a circuit that stumps me, I'll often draw a block diagram to start figuring out the algebra for the circuit.

CLICK We can use Black's formula on this block diagram to find the closed-loop gain, and as usually it's the forward path (1/rpi+gm)(ro||RE) over 1+loop gain, which is the same because of unity feedback. You can see that this expression is identical to the av we calculated earlier if you multiply the top and the bottom by rpi. So this feedback model is accurate the amplifier's operation. You can also use it to find rin and rout if you think carefully about what forward and feedback paths to use, but I'll leave that as an exercise for the reader.

One other thing to note is that our picture of the emitter follower doesn't look like it has feedback in it. There's no element wrapped around from our outut to our input. However, the base-emitter voltage gets shrunk by the current through the gm generator, which is a feedback loop. This type of feedback is often called series feedback (technically, series-series feedback because it's in series with both the input and the output), so keep your eyes peeled for it. I've seen series feedback on the ground node of a high frequency circuit cause instability, so it's good to know about.

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Summary

- Feedback is a powerful design tool that comes with a risk of instability
- A very common application of feedback is controlling gain.
- Controlled gains have a steady state error of $\epsilon_{ss} \approx 1/L$ where L is the loop gain.
- You can implement feedback in single-ended circuits like the emitter follower, which is well described by a feedback model.

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Impedance Modulation

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In this video we're going to look at the effects of feedback on the input and ouput impedance of an op-amp. We'll find that there's an apparent underlying link to control theory: the impedances will be boosted or reduced by a factor of 1+ the loop gain. We're not going to follow that thread all the way down into block diagrams, but it will be a useful rule for us to remember.

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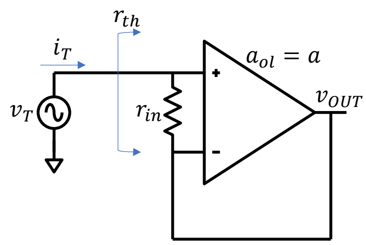
Boost r_{in} By Reducing Swing Across It



$$v_{OUT} = a(v_T - v_{OUT}) \rightarrow v_{OUT} = \frac{a}{1+a} v_T$$

$$i_T = \frac{v_T - v_{OUT}}{r_{in}} = \frac{v_T \left(1 - \frac{a}{1+a}\right)}{r_{in}}$$

$$r_{th} = \frac{v_T}{i_T} = \frac{r_{in}}{1 - \frac{a}{1+a}} = r_{in}(1+a)$$

Reducing swing with feedback boosts R by $(1 + L)$

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We're going to start by looking at this op-amp, which has a differential mode input rin that I've indicated between the + and – terminals. We're going to figure out what iinput impedance the driving source perceives, rth, which is probably going to be some function of the actual input impedance rin.

CLICK We start by finding the relationship between vOUT and vT, which requires us to do a quick rederivation of Black's formula. This expression also implicitly tells us the loop gain is just L=a. The expression also presumes the op-amp output has no output impedance.

CLICK We can use that fact to find the current iT. It's just going to be given by vT-vOUT over the resistor rin. You can substitute the earlier expression to get the current as just a function of vT, rin and a. I want to pause here to call attention to the fact that the feedback has dramatically reduced the swing across rin compared to a situation where the –ve terminal was grounded. That means way less current is going to flow into it than if all of vT were allowed to fall across rin.

CLICK Our Thevenin resistance is vT/iT, which we can find by rearranging the expression on the previous line. If you simplify the denominator, we find that our apparent input resistance, this rth, is 1+a times bigger than our actual input resistance.

CLICK The general principle that I take from this example is that if you use feedback to reduce the swing across a resistor, then its apparent value increases by a factor of 1+L. We can see that here because our swing was reduced and our loop gain is just a, so rin is bigger by a factor of 1+a.

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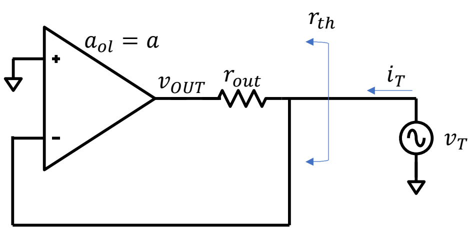
Reduce r_{out} By Increasing Swing Across It



$$v_{OUT} = a(0 - v_T) \rightarrow v_{OUT} = -a v_T$$

$$i_T = \frac{v_T - v_{OUT}}{r_{out}} = \frac{v_T(1+a)}{r_{out}}$$

$$r_{th} = \frac{v_T}{i_T} = \frac{r_{out}}{1+a}$$

Increasing swing with feedback reduces R by $(1 + L)$

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We can also find ways to modulate an op-amp's output impedance using feedback. Here's an example of testing the apparent output impedance, rth, of an op-amp with an impedance rout that is wrapped in a unity gain feedback loop. The unity gain feedback means the loop gain of this amplifier is just L=a.

CLICK vOUT is just given by –a times vT because the positive terminal of the op-amp is grounded.

CLICK and we can find the current iT using the difference across it, vT-vOUT divided by rout. This presumes the op-amp input has a big input impedance. We rewrite this in terms of just vT, a and rout. It's worth noting here that the op-amp is dramatically increasing the swing across rout because vOUT is swinging very far – remember it's value is a*vT – to try to cancel out the signal at the op-amps negative terminal.

CLICK rth is given by vT over iT as usual, and that's just rout/(1+a) based on the previous line. This is a big reduction in the apparent output impedance, and it comes from vOUT swinging wildly to try to force vT to be zero.

CLICK This principle generalizes. If we increase the swing across an element with feedback, we reduce the apparent resistance by a factor of 1+L.

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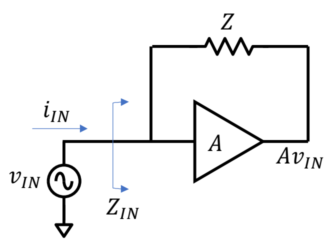
The Miller Effect is Single Ended Z Modulation



$$i_{in} = \frac{v_{in} - Av_{in}}{Z}$$

$$Z_{IN} = \frac{Z}{1 - A}$$

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We've already seen an example of impedance modulation, which was the Miller effect. I wanted to revisit our Miller analysis to show a few things things. First, to show again that we can make feedback in single-ended amplifiers, we don't need differential inputs for feedback. Second, to show another type of impedance modulation, which is decreasing input impedance with feedback. Third, to show that our dynamics analysis ties into feedback too.

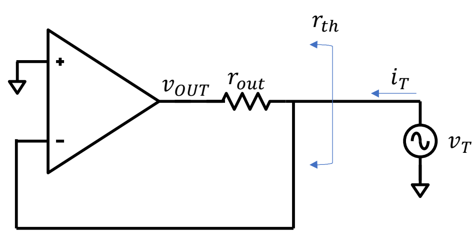
This picture shows an ideal single ended amplifier with gain A and an impedance in feedback around it. That means the output voltage of the amplifier is just equal to A times the input voltage, which increases the swing across the element Z.

CLICK We can find the input current using the difference between vIN and vOUT then dividing by Z. And we can rearrange that expression to find our apparent input impedance ZIN.

Our earlier input impedance example showed how to increase input impedance, but this shunt feedback configuration does the opposite. This increases the swing across our resistor to create an input impedance that is decreased by 1+ the loop gain. That statement might give you pause because we have 1-A in the denominator instead of 1+A, but don't forget that our feedback diagram has a negative sign on the summing junction, which we are lacking here because the amplifier has a positive gain. If we make A a negative number, then this loop gain factor will look like a standard negative feedback. So, as we saw earlier: if feedback increases swing then we reduce impedance by a factor of 1+L.

Summary

- Feedback can be used to boost or decrease input and output impedance.
- The impedance is boosted or decreased by a factor of $(1 + L)$
- Eg:



$$r_{th} = \frac{v_T}{i_T} = \frac{r_{out}}{1+a}$$

The loop gain is $L = a$

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Stability and Phase Margin

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In this video we're going to develop a tool that we can use to identify whether an amplifier is stable. This tool, called phase margin, is very practical and easy to measure in lab, but we make a few approximations to get to it. Don't forget to use it in conjunction with other stability analysis tools like root-locus, Nyquist plots, or common sense.

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What is Instability?

- Bounded-input, bounded-output less useful in circuits b/c everything bounded by rails. (But watching for poles in RHP is still useful.)
- Formally, concepts from non-linear control help (limit cycles, Lyapunov stability functions, describing functions)
- Informally, only get signals we put there + harmonics.
- Sneaky feedback through supply / parasitics reduced by decoupling.

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One of the first challenges of analyzing the stability of amplifiers is deciding on what stability even means.

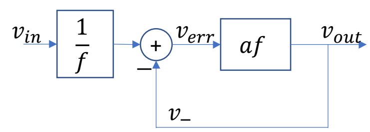
CLICK The typical definition given in controls classes is bounded-input, bounded-output stability. However, all outputs from circuit we've seen are bounded by the supply rails, so this definition loses some steam. Fortunately, our linearized small signal holds when our output is far from the rails, so the condition we used for stability of linear systems – no poles in the right half plane – still helps us identify when our signals are likely to get big and weird.

CLCIK This bounding by rails is a non-linear behavior that our amplifiers engage in as they fall out of forward active near the rails, so linear control theory stops working well for them. Fortunately, non-linear control has some answers for us. You can look up limit cycles, Lyapunov stability functions and describing functions if you want to get a handle on the formal mathematical descriptions of non-linear amplifiers.

CLICK However, we're not doing that. Our, much coarser, description of stability is that we can account for all the signals at the output. They're either at frequencies we put in, or they're harmonics of those signals. If a signal comes out of nowhere, especially if there's no input, then we are going to call an amplifier unstable. That definition is pretty straightforward, which is good news. One other piece of good news is that

CLICK We often see instability when we're building open-loop amplifiers in the lab. For instance, really high gain common emitters with long power supply wires can oscillate. That's because there are sneaky, parasitic feedback loops in the lab, that couple through the power supply, biasing networks or other parasitics. These parasitic feedback loops can cause instability even when you don't add explicit feedback around amplifiers.

Unstable if Poles in RHP, $L(j\omega) = -1$ is Hint



$$a_{CL}(s) = \frac{1}{f} \cdot \frac{L(s)}{1 + L(s)}$$

Closed loop gain is system transfer function, so unstable if $1 + L$ has positive roots.

Often know roots of L, but finding roots of $1 + L$ is pretty tough!

$$a_{CL}(j\omega) = \infty \leftrightarrow L(j\omega) = -1$$

We know a pole is on the imaginary axis if $L(j\omega) = -1$

We don't know if that pole is moving left or right as L increases (Barkhausen issue)

BUT if $\angle L$ is monotonic, this probably indicates a transition from stable → unstable

$$L(j\omega) = -1 \rightarrow |L(j\omega)| = 1$$
$$\rightarrow \angle L(j\omega) = \pi$$

Express this constraint as magnitude and phase, then pick off from Bode plot of L!

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We're going to start developing a stability criterion for our transfer function by asking "why is it so hard to tell if a system is stable"? We are considering this feedback system to answer that question, and we notice that this system has the transfer function indicated on the left.

CLICK The first thing to note is that this closed loop gain is a transfer functions, so if any of the roots of 1+L(s) are in the right half-plane then it will be unstable. OK, that's fine, but finding the roots of 1+L(s) is mathematically tough and not intuitive.

CLICK That's frustrating because we usually know the poles and zeros of L(s), but adding one to it gives us a whole new polynomial to factor. So we're hoping to figure out if we're stable just by looking at L(s).

CLICK We're going to start that by noticing that L(jw)=-1 if and only if there is a pole on the jw axis. Said another way, a pole on the jw axis results in the closed loop gain blowing up to infinity, and that means the denominator of aCL has to be zero.

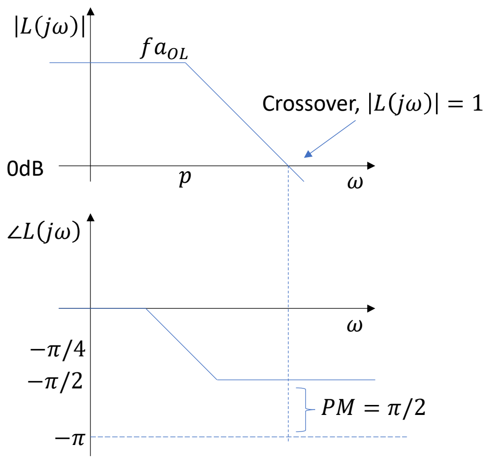
CLICK That's an interesting fact, but we don't know for sure if the pole that is currently on the axis will move onto the RHP or the LHP as L increases. If it moves to the right, then L(jw)=-1 is a stability criterion, and that criterion is often called the Barkhausen Criterion.

However, poles can move either right or left depending on the specifics of the root-locus, so you can't actually count on Barkhausen all the time.

CLICK That said, if we make a few assumptions about L, specifically that the phase decreases monotonically, then L(jw)=-1 indicates a boundary between stability and instability.

CLICK So we express this constraint in terms of magnitude and phase of L(jw), and we also note that we can plot L(jw) on a Bode plot pretty easily. If we do that, then we should be able to identify where the magnitude is 1 and the phase is pi pretty easily.

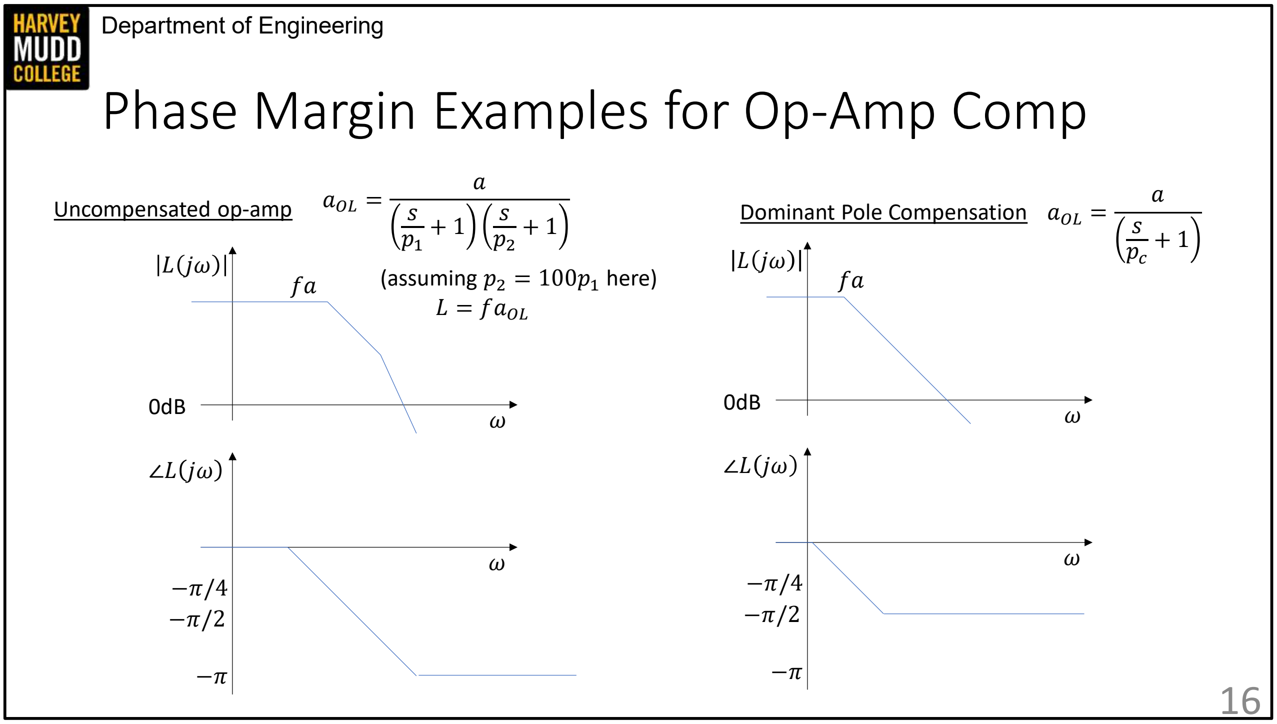
Phase Margin: $\angle L(j\omega)+\pi$ when $|L(j\omega)|$ is 1



- If phase margin is zero, then the system is unstable.
- Phase margin is a relative measure, more overshoot as it approaches zero.
- Changing f changes location of crossover.

So here's a Bode plot of loop gain for a first-order L(jw). We see the magnitude slope turn down at the pole location p and continue to decrease forever. At some frequency, the magnitude is equal to 1, or to 0dB, and that point is called crossover.

The phase margin is the difference between the system's phase and pi at crossover. It's indicated graphically on the phase plot, and it's pi/2 in this case. If the phase margin is zero, then we're at the border of stability and instability per the derivation we came up with on the other page. We're assuming a little bit about the behavior of poles, that they are moving from left to right in the root locus, by reading more into the phase margin. But if we do so then we can say that the system is unstable if phase margin is negative, stable if it's positive, and that decreasing phase margin leads to more overshoot as the poles approach being purely imaginary.



We’re going to look at a few examples of loop gains to get a feel for phase margin. We’re going to start by looking at an uncompensated op-amp transfer function. Here we’re assuming that the gain stage provides the two most important poles in the op-amp, which is often true. That means we have two poles in our denominator. We’re ignoring the feedthrough zero for simplicity. Two other details are important. We’re assuming that the second pole is a factor of 100 above the first, and we’re assuming this op-amp is in feedback such that the loop gain is the feedback factor times the open-loop gain.

I’m going to show you the magnitude plot for this function in a second. Pause the video and sketch the general shape of this magnitude plot, but don’t worry about getting it exactly right because I haven’t given you all the information you need to get the details right.

CLICK This is a two pole transfer function, so it starts at the value of fa, then turns to -20dB per decade decrease, and finally to -40 dB/decade after the second pole. I didn’t give you a value for fa, so your crossover might be at a slightly different spot than mine.

Next we’re going to drawn the phase plot for this specific magnitude plot that I’ve shown and assess its phase margin. You have enough information to make a pretty good drawing of the phase, so try to get the phase margin current, though I acknowledge that the lack of gridlines and numbers makes that job harder. Pause the video and try to drawn the phase plot and guess the phase margin.

CLICK The phase plot will decrease continuously from 0 to pi over four decades because the poles are separated by a factor of 100. The first pole causes the phase to decrease by pi/2 over two decades, then the second pole kicks in and moves the phase another pi/2 over the next two decades. The phase margin for this op-amp is going to be very small or equal to zero, so this amplifier is barely stable if it is stable at all.

CLICK Now we're going to try to find the phase margin of a dominant pole compensated op-amp. Here, we model the transfer function with one dominant pole. You might wonder where the second pole went. One argument I could make is that the dominant pole is really low now, so we're likely to cross over before the second pole matters. That would hold water, but it turns out that dominant pole compensation also pushes the second pole to a higher frequency because the compensation cap modulates the output impedance of the gain stage. So a one pole model works really well for a dominant pole compensated op-amp.

Pause the video, draw the Bode plot for uncompensated loop gain, and find the phase margin for this amplifier.

CLICK The really low frequency pole causes our gain to roll off early, but because a is big, it still usually takes a few decades to reach crossover. However, our phase stops at –pi/2 because we only have one pole. That means our phase margin is pi/2 and this amplifier is very stable.

Summary

- For us, Instability is getting a signal at the output we didn't put there.
- Feedback through parasitics or supply coupling can cause instability.
- Phase margin is $PM = \angle L(j\omega) - (-\pi)$ when $|L(j\omega)| = 0$dB. Pick off from phase value at crossover on a Bode plot.
- Phase margin measures stability if poles are moving from LHP to RHP. $PM > 0$ is stable, closer to 0 is less stable.

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Gain-Bandwidth Product

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In this video we're going to look at the gain-bandwidth product, which is a commonly listed specification on many op-amp datasheets. This spec says that the product of the closed loop gain of an op-amp and the bandwidth in that configuration is always constant. This relationship is so ubiquitous that it's easy to think of it as a physical law, but the gain-bandwidth product actually comes from dominant pole compensation. We'll show that in this video.

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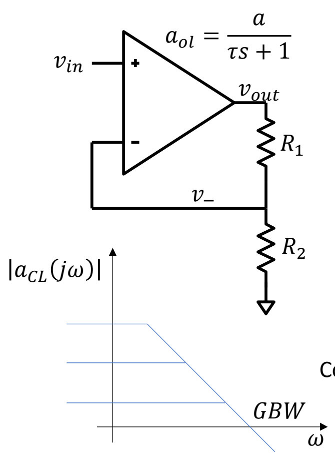
Const. GBW Product Comes From 1 Pole a_{OL}



Increasing G and decreasing $1/\tau_{CL}$

$$a_{CL}(s) = \frac{1}{f} \cdot \frac{\frac{af}{\tau s + 1}}{1 + \frac{af}{\tau s + 1}}$$

$$= \frac{1}{f} \cdot \frac{af}{1 + af + \tau s}$$

$$= \underbrace{\frac{1}{f}}_{\text{Commanded gain}} \cdot \underbrace{\frac{af}{1 + af}}_{\text{Static error term}} \cdot \underbrace{\frac{1}{1 + \frac{\tau}{1 + af} s}}_{\text{Dynamics (“dynamic error”)}}$$

$$G = \frac{1}{f}$$

$$BW = \frac{1}{\tau_{CL}} = \frac{1 + af}{\tau}$$

$$GBW = \frac{1}{f} \frac{1 + af}{\tau} \approx \frac{a}{\tau}$$

Approximately constant because of how dominant pole compensation moves τ

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We’re going to look at a standard op-amp in non-inverting feedback like we’ve done in the past few videos. One change we’re making is to assume the op-amp is dominant pole compensates, so it has a single pole located at 1/tau.

CLICK The closed loop gain is given by the forward path over 1+ the loop gain, and I’ve pulled our commanded gain of 1/f out in front of the closed loop gain expression so it’s L/(1+L).

CLICK We can clear the fractions from this expression

CLICK And then factor it into a instructive form. Here we have our commanded gain, multiplied by a steady state error term and finally by a first order dynamics term. The dynamics term is sometimes called the dynamic error because it will approach 1 on a long enough time scale. It’s worth noting that the location of the closed-loop pole is determined by af because the coefficient in front of s, which is an “closed loop” time constant, has af in it.

CLICK So that means our commanded gain is 1/f. By the way, this context is one where the distinction between commanded gain and closed loop gain gets really confusing. Lots of people call the gain of the op-amp circuit the closed loop gain, but we’re sticking with commanded gain to separate this from the transfer function. The closed loop bandwidth is given by one over the closed-loop pole, so it's the quantity 1+af over tau. Multiplying those quantities gives us something that is almost independent of gain (if we assume af is much bigger than 1, which is usually true). So that means the product of commanded gain and closed-loop bandwidth is a constant in this op-amp.

That's a useful fact when you're designing with dominant pole compensated op-amps. However, if you use other kinds of compensation to push the performance of the op-amp you can break this tradeoff. Constant gain-bandwidth products come from dominant pole compensation, and other open-loop transfer functions would have other behaviors.

CLICK By the way, there's a neat graphical way to see a constant gain bandwidth product in action. We know that the closed loop gain of this amplifier increases linearly as 1/f gets small, but that causes the bandwidth to shrink linearly because af becomes smaller. As a result, we get this neat family of transfer function for different amounts of gain, where the pole is at a different spot, but the first order rolloffs all line up at high frequencies. The crossover for this whole family of curves occurs at the gain-bandwidth product because that's the spot where G=1/f is equal to 1. By the way, the fact that 1/f is equal to 1 here means that the system is in unity gain feedback, which is the op-amp configuration with maximum bandwidth.

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Summary

- Constant (commanded) gain, (closed-loop) bandwidth products come from dominant pole compensation.
- This has a graphical interpretation, where GBW is the crossover of the family of closed-loop transfer function curves.