

In this video series we're going to discuss output stages, which are amplifiers that can drive high currents or small loads. As the name suggests, they're often used as the outputs of op-amps. Op-amps have to drive all sorts of weird loads, so a good output stage is an important key to making an op-amp useful. Next video series we're putting this together with differential amplifiers to build our first op-amp.

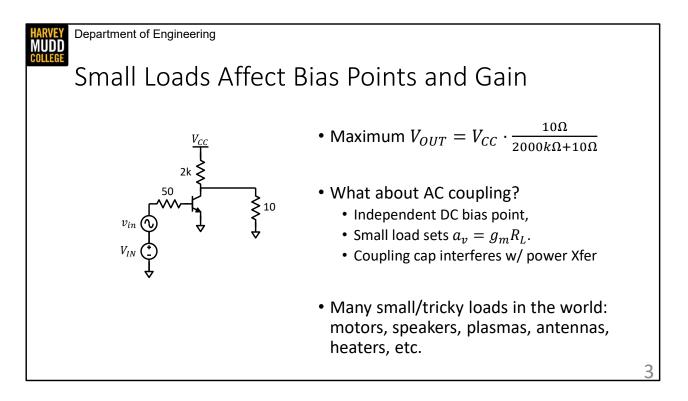


Department of Engineering

Why Use Output Stages and Max Power Transfer

Matthew Spencer Harvey Mudd College E151 – Analog Circuit Design

We haven't used output stages yet this semester, so why start now? This video will go over a quick justification for output stages, which are also sometimes called power amplifiers, and then cover an important theorem that lends us insight about how to transfer power into a load. Appropriately, the theorem is called the max power transfer theorem.

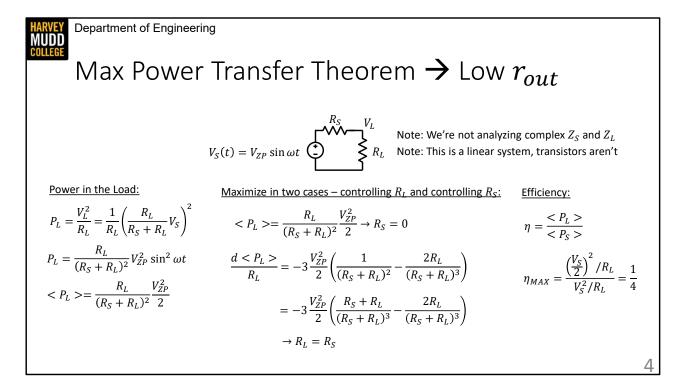


I've drawn a common emitter here that is DC coupled to a very small load resistance.

CLICK As you might have discovered in lab, very small load resistances mess with your bias point. In this case, we know that the highest voltage we could possibly get at the output node will occur when the transistor is in cutoff, and that output voltage still won't be very high. In fact, if that load is small enough, then the transistor might never be able to leave the saturation region.

CLICK OK, so what if we AC couple our loads. That's how we've been measuring output resistance, why wouldn't it work here? It would, in the sense that our collector's DC value would be whatever we had originally intended. However, the small load would still limit our gain, so it's not clear that we would be faithfully driving the vin signal onto the load. Also, the coupling cap makes power transfer frequency dependent and difficult, which we will start to inspect on the next slide.

CLICK It's worth noting that small loads aren't an academic problem: there are lots of tiny, weird loads in the world like motors, speakers, plasmas, antennas heaters and others.



Because there are so many weird loads in the world, it's worth making a unified theory to figure out how to get power into them. That tool is called the maximum power transfer theorem. We derive it using the circuit pictured here.

CLICK Before we begin, I need to make two caveats. First, we're going to do this analysis using real resistors because that's the most familiar material in this class. However, there's a similar analysis that tells us a bit more information about reactive loads. Second, we're analyzing a linear system here, but we build amplifiers out of transistors, which are wildly non-linear. Even worse, we can't use our usual dodge of retreating to small signal analysis, because we want output stages to deliver high power, which means they need to have big voltage and current swings. However, you can still get a little bit of insight and/or ballpark estimates from small signal analysis at the extremes of an output stage's operation, and accordingly we can get a bit of insight or intuition from the max power transfer theorem.

CLICK We start by figuring out how much power is dissipated in the load. Because the load is resistive, that's given by the load voltage squared over RL. The load voltage is expressed as a voltage divider multiplying VS.

CLICK We can substitute the value for VS and simplify some of the resistances out in front. Doing so reveals that the power is a function of time, specifically a sine squared. CLICK We almost always care about the time averaged power instead of the instantaneous power. When we take a time average (which I've indicated with brackets), the sine squared turns into a factor of one half, which is a fact you should memorize.

CLICK Awesome, we have this expression for PL, let's see what we can do with it. We know that we want PL to be as big as possible, but there are two scenarios we have to consider. In the first, we control the source resistance. Just looking at the PL expression, we can see that RS only appears in the denominator, so we should minimize the source resistance to maximize the value of PL

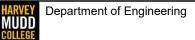
CLICK The situation where we control RL is trickier to figure out. We have to take a derivative because RL is in both the numerator and denominator of our expression. I've done so here.

CLICK We can simplify that expression a bit in order to get everything over the same denominator. That means we can reduce our problem to finding RL such that these numerators, RS+RL-2RL, add to zero.

CLICK That's easy enough algebra, and it tells us that RL is equal to RS. Awesome, so now we know both how to send and receive power. Since we're building amplifiers, we're mostly in the case where we control the source, so the big insight we can get here is that we should design amplifiers that have very low output resistances. Now, the fact we're dealing with large signals complicates that message a bit since output resistance changes with bias point, but it's a good starting point for figuring out what an output stage might look like.

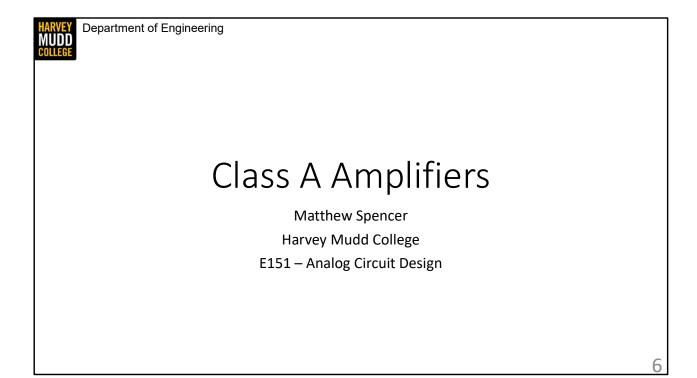
CLICK One final note, efficiency is a very fundamental measure of power transfer. It's the ratio of power delivered to the load divided by power pulled from the source. You might wonder what the maximum efficiency of matched power delivery looks like.

CLICK The answer is pretty bad. Because RS is equal to RL when we have maximum transfer, we know the voltage at the load is half the source voltage. Squaring that gives us an efficiency of 1/4.

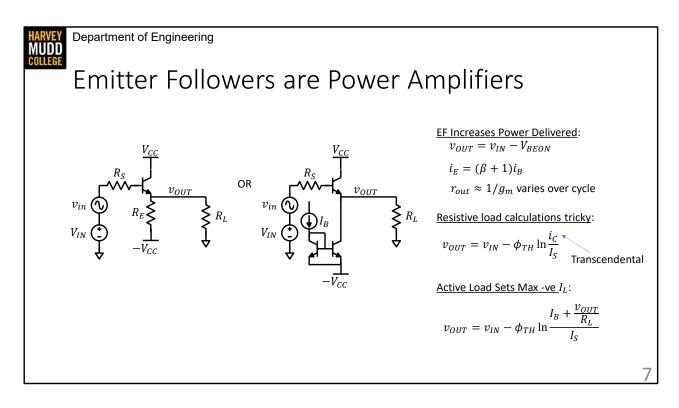


Summary

- We need special circuits to drive small loads, strange impedances, or lots of current. These are called power amplifiers or output stages.
- Maximize power transfer by minimizing R_S or matching R_L .
- Efficiency is $\eta = P_L/P_S$.
- High power circuits require large signal analysis.



In this video we're going to talk about the simplest type of output stage, which is called a class A amplifier.



Class A amplifiers is a very fancy name that almost always just refers to an emitter follower. It has a few extra implications, but not that many. We'll talk about them on a later slide.

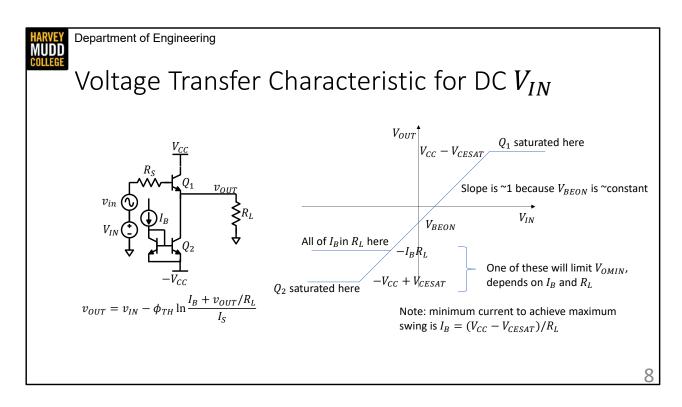
I've drawn two emitter followers here, one with a resistive bias and one with a current mirror bias. I've also chosen to power this emitter follower with split rails, both a positive VCC and a negative VCC. The negative VCC rail is often named VEE, but I've just put its voltage value there to simplify our notation. The split rails let us pick the DC bias VIN so that no power is dissipated in RL when the signal indicated by little vin is zero. I'm calling that signal little vin instead of small signal vin because it isn't really a small signal here. We're using small signal notation to indicate that little vin isn't constant, but it will swing basically rail to rail if we're trying to deliver power to RL.

CLICK Emitter followers have a small rout value, so we might have an instinct that they'll be good power amplifiers based on the max power transfer theorem. We need to do some large signal analysis to confirm that suspicion. First, we note that vOUT is just our total vIN signal minus VBEON. That means the amplitude of the voltage waveform on vOUT is the same as the one on vIN. However, the current driven into the output is going to be about beta times higher than the current driven into the base. That means we get the same voltage at a much higher current, which increases the power at our output. Finally, it's worth noting that the reason our initial suspicion wouldn't work is because rout changes as vIN moves around because rout is proportional to 1/gm and gm depends on iE.

CLICK The resistively biased emitter follower is relatively unpopular because it's tough to analyze and, as a result, tough to design. You'll see something that looks like that circuit in a lot of places, notably in voltage regulators where vOUT is controlled by feedback. (Also, voltage regulators take RE out of the circuit and use RL as the DC path to ground, which I find clever.) However, looking at a detailed calculation of the vOUT voltage where VBEON has been replaced by phi_TH*log(iC/IS), we realize this is going to be a brutal equation to solve. Notably, figuring out iC is a transcendental resistor-diode problem.

CLICK The active load sidesteps that problem by pinning the total current that flows through the transistor in a convenient way. Finding vOUT is still a transcendental equation, but we can make some observations about what happens at high and low extreemes of vOUT, notably that when vOUT/RL is equal to –IB we're going to see vBE start to get really big.

... show RL calculation with negative vIN on resistive load?



One easy way to start understanding the large signal behavior of this amplifier is drawing the voltage transfer characteristic or VTC, which you will recall is a plot of large signal VIN vs. large signal VOUT. I've put the axes over on the right to start doing that. I've also included the equation describing vOUT from the last slide for reference.

CLICK We can start by noticing that the VTC will basically be a straight line for much of the curve. In this region IB is bigger than vOUT/RL, so the logarithmic argument in the vOUT equation is basically constant. This line has a value of zero when VIN is equal to VBEON. That's because of the VBEON drop of Q1. Picking large signal VIN=VBEON is a pretty common choice because then our output doesn't dissipate any power when little vin is zero.

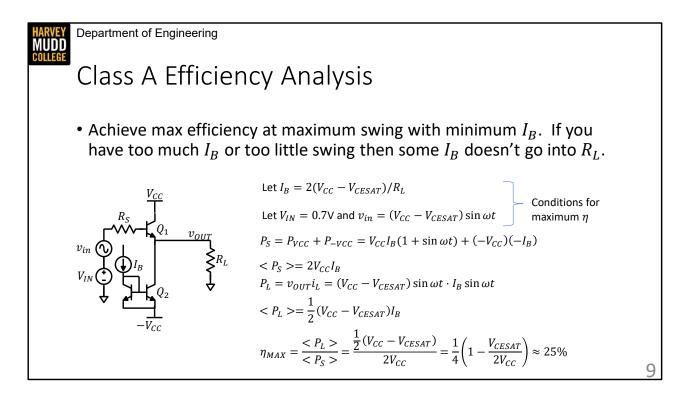
CLICK In the case that RL is big, the vOUT/RL term never gets close to IB, so the transfer characterstic has a slope very close to 1. We'll see that behavior change when the amplifier stops working because either Q1 or Q2 saturates, which happens when the output gets within VCESAT of the rails. Hitting the Q1 failure mode is unusual because it requires VIN to swing above VCC. The input is VBEON above the output, so when the input is at VCC, the output is at VCC-VBEON, which is still in the linear region of the amplifier's operation.

CLICK However, if RL is not very large then there's another possible failure mode. If the current in Q1 goes to zero then it's cut off, so Q1 will be unable to convey additional changes in VIN to the output. That happens at a voltage of –IB*RL because all of the current flowing into Q2 has been steered into RL at that point. (You can imagine the purpose of Q1 in this circuit as stealing some of the Q2 current from RL to increase the output voltage.) This causes the output voltage to be pinned at its lowest value, -IB*RL. This appears in the vOUT equation because the log function represents vBE and the log function becomes very large as –vOUT/RL cancels out IB.

CLICK That means we have two possible conditions that set the minimum output voltage of the amplifier. Which one dominates depends on the values of IB and RL, which you can anticipate as a designer. One desirable condition is to have these two conditions occur at the same voltage so that you're neither constrained by your bias current, nor paying extra power for current you don't need. That occurs when IB*RL = 2(VCC-VCESAT).

... IB*RL matches with VBEON getting big. (Sat is different effect)

... Note that you need your input to swing outside the rails to reach Q1 saturation



It's common to use efficiency as the main figure of merit for power analyzers, so I want to show you one or two efficiency analyses in these videos to familiarize you with the how efficiency is calculated. However, we're not going to do a lot of efficiency analysis on our own in the lab, so enjoy the show and try to remember rough outlines of the process. You can just write down the results at the end of the page.

CLICK When you analyze efficiency, you usually do it in the most favorable condition for the amplifier. For this power amplifier, the most favorable condition is when the voltage swing is at its maximum value. That's because the IB current is always flowing, whether you are driving a large output voltage or a small one. So we want to drive a big output voltage to use up all of the IB current. For the same reason, we want to make sure IB is as small as possible to achieve our maximum swing.

CLICK That gives rise to these test conditions. We're setting IB to achieve maximum swing, which is a value we calculated on the last slide, we're setting large signal VIN so that our output is zero centered, and we're picking little vin to be a sinusoid with the maximum possible swing of VCC-VCESAT.

CLICK The power pulled from the supply is made up of power pulled from VCC and power pulled into –VCC. The power pulled into –VCC is easy: Q2 is always sinking IB of current.

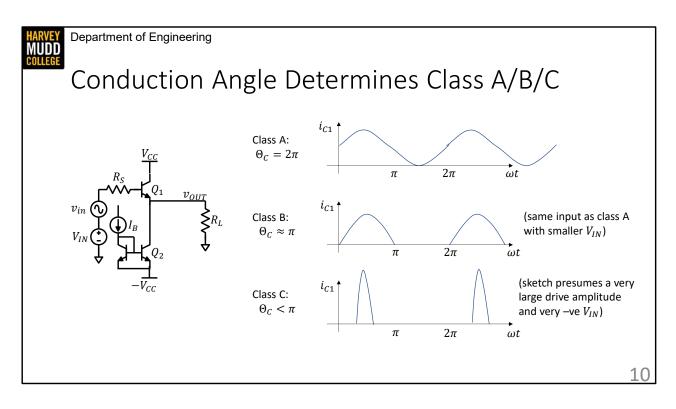
The power from the positive supply is given by VCC times IC of Q1, which varies sinusoidally with the input. When vin is zero, that current has to be IB. When vin goes to its max value, we pick up another (VCC-VCESAT)/RL of current, which I've chosen to write as IB here.

CLICK Time averaging this, we see that PS is 2*VCC*IB. This makes sense: on average IB runs from +VCC to –VCC.

CLICK The power delivered to the load is a bit more complicated because some power gets dissipated in Q1. The easiest way to figure it out here is multiplying vOUT by iOUT. We've set up our input voltages so that vOUT is VCC-VCESAT times sine omega t. Our current is just going to be proportional to that because our load is a resistor, but we've chosen to write the current coefficient as IB instead of (VCC-VCESAT)/RL.

CLICK Time averaging the load power turns the sine squared into a factor of 1/2.

CLICK Finally, we take the ratio of load power to source power to find our efficiency. The IB values cancel out, which is why we kept writing stuff in terms of IB above. We don't get particularly good news: the efficiency peaks at about 25%, which is quite bad! This is also our maximum efficiency, and it gets worse as we back off from maximum swing. The main cause of this is the fact we're always dissipated IB from our supplies to keep Q1 in forward active.

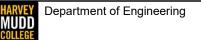


So we might wonder if we can get away with not running current in Q1 all the time. That line of thinking leads us to some other very common amplifier topologies called class B and class C amplifiers. The circuit on the left can represent any of class A, B or C amplifiers, the only difference between them is the value of VIN. More specifically, the way you determine if an amplifier is class A, B or C is a parameter called the conduction angle, which I've given the symbol capital ThetaC. The conduction angle describes the portion of the input wave's period during which Q1 is conducting current. In a class A amplifier, the conduction angle is 2*pi, which means it conducts current during the whole cycle of the input wave. You can see in the top-most plot of Q1's collector current vs. omega*t, which is the usual way to look at conduction angle. Using omega*t on the x axis let's us talk about fractions of a period without paying attention to frequency, which is where the concept of a conduction angle comes from. iC1 is always greater than zero, so its conduction angle is 2pi. Class B amplifiers have conduction angles that are close to pi, and class C amplifiers have conduction angles that are much less than pi. Class C amplifiers are also usually driven very hard, which is why I've made the current spikes in the class C graph very tall.

You might look at the class B and class C amplifier outputs and think they look nothing like our input sine wave. How can those possibly be useful amplifiers if they throw away so much information about our wave? There are circuit tricks that let us use both class B and class C amplifiers to replicate the input pretty faithfully, and these tricks include push-pull configurations and resonant output filters. More on thoses later.

... iC1 can't be negative, current only flows down through Q1, so you can think of Q1 canceling effect of IB on RL

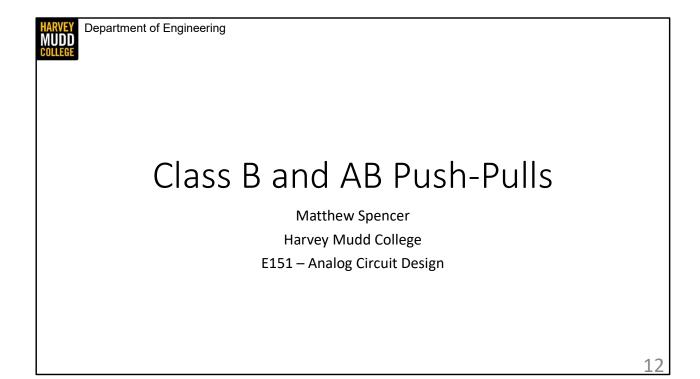
... class B and C have high distortion right now. Can use output filters. We're also going to see a class B variant that does much better.



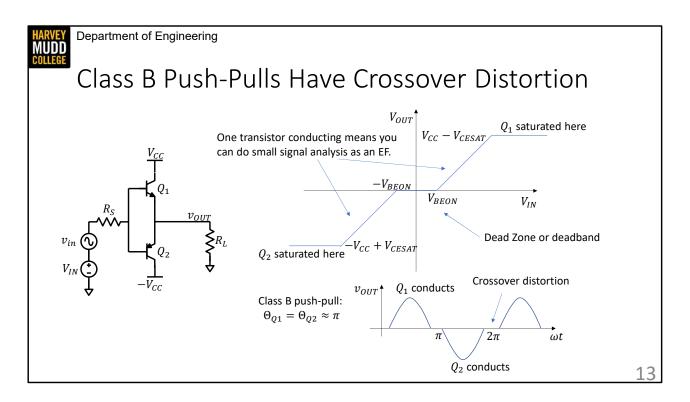
Summary

- Emitter followers are power amplifiers with $V_{OMAX} = V_{CC} V_{CESAT}$ and $V_{OMIN} = \max(-I_B R_L, -V_{CC} + V_{CESAT})$
- Operated as class A amplifiers (2π conduction angle) at maximum swing, class A amplifiers achieve 25% efficiency.
- Backoff degrades efficiency because I_B is running all the time.
- Class B and C amps are the same topology with less conduction angle.

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In this video we're going to look at different types of power amplifiers called push-pulls. Push-pulls are very good amplifiers, and they are the most popular linear power amplifiers in use today.



Our class B amplifier had a conduction angle of pi, which meant that we had really bad distortion at the output. One fix for that issue is to make a class B push-pull stage, which is pictured on the left. This kind of looks like two emitter followers, one NPN and one PNP, stacked on top of one another. Push pulls are set up to have Q1 conduct for pi radians while sourcing current to RL and then to have Q2 conduct for pi radians while sinking current from RL.

The axes for a transfer characteristic are set up pictured in the upper right.

CLICK We can start filling in the VTC by noting that when VIN is close to zero, neither Q1 or Q2 is turned on so VOUT doesn't change when VIN does. That behavior is called a dead zone or a deadband.

CLICK Once Q1 or Q2 turns on, that device acts like an emitter follower until it saturates. Just like class A amplifiers, reaching these saturation points it unusual because it requires VIN to swing outside of the rails.

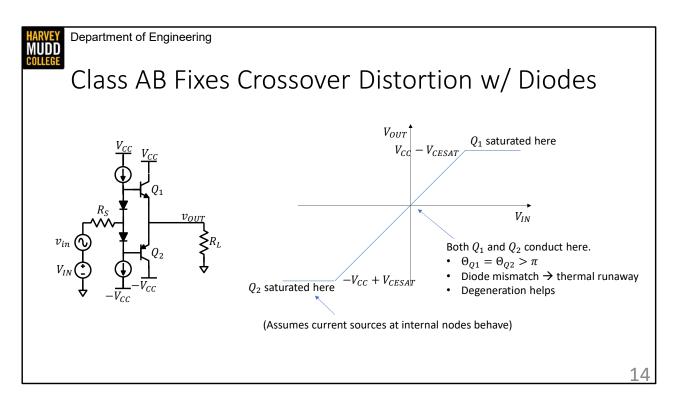
CLICK One side note, if you need to do small signal analysis for some reason – maybe estimating peak output impedance or something – you can just treat the circuit as an emitter follower in these zones because one of the transistors is in cutoff. RL is going to act

like the emitter resistor.

CLICK This is a weird voltage transfer characteristic, and it leads to a weird output wave. Specifically, our amplifier stops responding whenever the input crosses zero, which is called crossover distortion. That's bad, but you might not care if your input signal is sufficiently large. It's worth noting that both Q1 and Q2 have conduction angles that are close to pi, which is why this called a class B push-pull. Q1 conducts during the positive half-cycle and Q2 conducts during the negative half-cycle.

... Crossover distortion

... Nice because biasing is easy

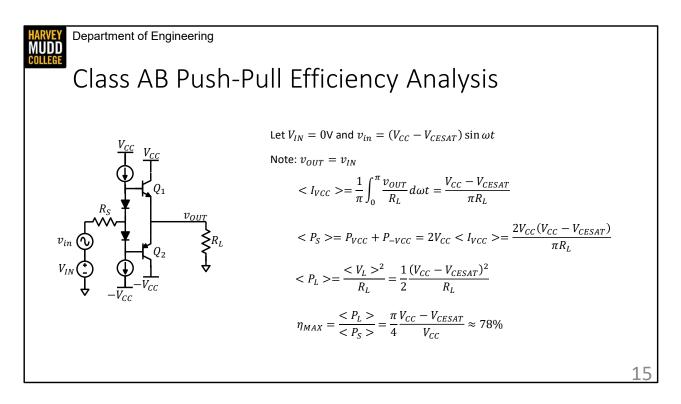


It's common to fix crossover distortion using a topology called a class AB push-pull. I've drawn one of those on the left. The big idea in a class AB push pull is that the on-voltage of the diodes cancels out the VBEON voltages of Q1 and Q2. So, for example, the base node of Q1 has a voltage of VON on it when VIN is zero, which lets Q1 respond instantly if VIN increases above zero. That gives rise to the voltage transfer characteristic I've drawn on the right.

CLICK There are a few details to note about this plot. First, pedantically, this Q2 saturation condition depends on the base node being able to swing outside the rail, so the current sources that keep the diodes turned on need to be very well behaved.

CLICK Second, there's a narrow window in the middle of this curve where both Q1 and Q2 are conducting. By definition, that means both Q1 and Q2 have conduction angles greater than pi, which is why this is called class AB, it's part way between the 2pi conduction angle of class A and the pi conduction angle of class B. It also means that we're susceptible to a cool circuit failure mode called thermal runaway. Thermal runaway comes from the fact that VBEON is dependent on temperature, so if a slight mismatch in the diodes makes Q1 take more of the current during this crossover, then Q1 will get hotter than Q2, that means the vBE of Q1 will shrink, so that it takes even more current on the next cycle. Eventually Q1 handles all of the output current and lights itself on fire. Fortunately, you can avoid this

by putting small resistors in series with the Q1 and Q2 emitters and with the biasing diodes.



Class B and class AB amplifiers have better efficiency than class A amplifiers, and we're going to go through a derivation to show that. Like the other efficiency analysis, we're mostly interested in the results and a chance to see the process. This analysis is for class AB amplifiers, but it's pretty accurate for class B amplifiers too.

CLICK We set large signal VIN to zero so that we don't dissipate DC power and we set little vin to (VCC-VCESAT)*sin(wt) to ensure that we're driving this amplifier at maximum swing. We'll see soon that our efficiency is still best when our swing is as large as possible.

CLICK I want to pause for a moment to note that if everything is working right in this amplifier, then our output voltage is equal to our input voltage. The base of Q1 is at vin+VON and the output is back down to vin+VON-VBEON, and VON cancels out VBEON to leave vout=vin.

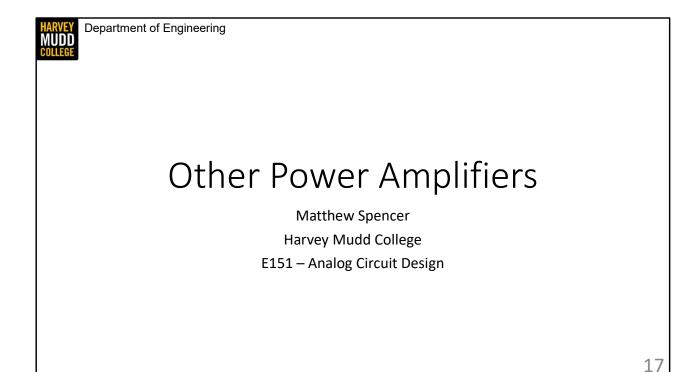
CLICK We're going to care how much current is drawn from the supply, so I'm going to start by calculating it separately here. We're want to average our current, which is given by vOUT/RL, and we're going to do so using the conduction angle instead of absolute time. Q1 only conducts from 0 to pi (more or less), so we integrate vOUT from 0 to pi with respect to omega*t. Because we're finding an average, we need to divide by one over the period out in front of the integral. Integrating a half-period of sine gives us 1, so our output is (VCC-VCESAT) / (pi * RL).

CLICK The power pulled from the supply is the power pulled from VCC plus the power sunk into –VCC. The power pulled from VCC is VCC times the current we just calculated, and the current sunk by –VCC is an exactly symmetrical calculation using –VCC and the current through Q2 in the negative half-cycle. So our result is 2*VCC*(the time average of ICC) or this value shown on the right.

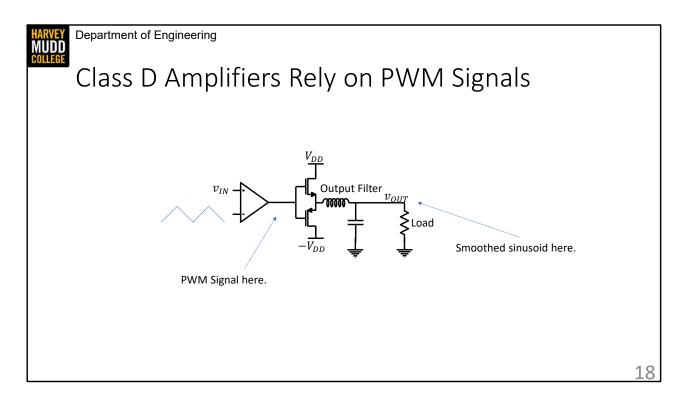
CLICK The power in the load is easier because it's a resistor. vOUT squared has a sine squared in it, and time averaging that gives us a factor of 1/2.

CLICK Finally, we find efficiency by taking the ratio of average load power to average source power. This comes out to pi/4 times (VCC-VCESAT) / VCC. If VCESAT is small, this result is about 78%, which is more than three times better than our class A amplifier. We also back off more gracefully because our IVCC scales with the input amplitude. However, that current is still pulled form a full VCC, so we are most efficient when we are pulling as much current into the load as possible.

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Summary
Push-pulls use two devices, one to source current for about half the cycle and one to sink current for the other half.
Class B push pulls have crossover distortion.
Class AB push pulls used diodes to cancel crossover distortion, but have a risk of thermal runaway.
Class B and class AB achieve peak efficiency of ~78%

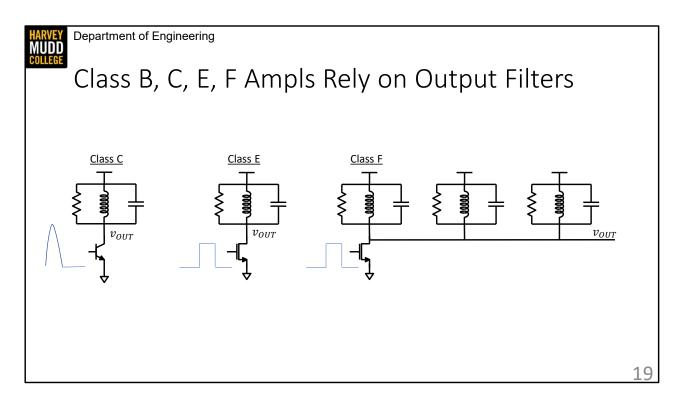


In this video we're going to do a quick survey of a few other power amplifiers. We're not analyzing these, I just want you to see some of the other technologies that are out there.



Class D amplifiers work quite differently from class A, B or C amplifiers. They work by driving a PWM signal onto a pair of switches, then filtering the output of those switches with a lossless LC output filter. That means the average power delivered to the output is going to depend on the PWM value, but the filter is going to force the PWM signal into a smooth shape that matches vin, usually a sinusoid. These are popular in audio applications because you can run the PWM at very high frequencies compared to the relatively low audio frequencies contained in the vIN signal. However, hardcore audiophiles are skeptical of class D amplifiers because they claim they can hear faint artifacts of digital modulation.

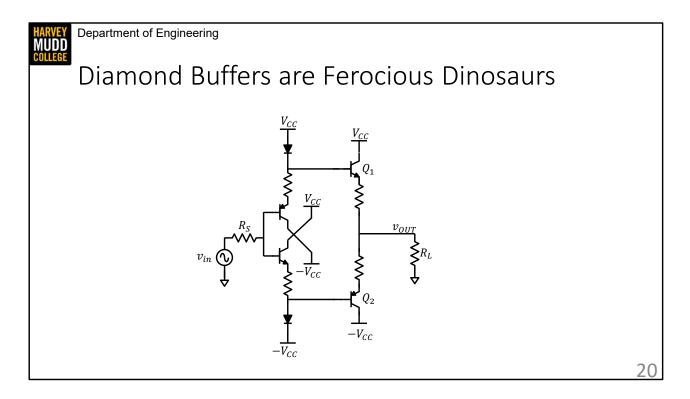
By the way, the op-amp on the left of this figure shows a neat way to make a duty cycled output. By driving a triangle wave into one input of the op-amp and comparing it to our input signal, we get a square wave at the output with a width that is proportional to vIN. That's because the time at which the triangle crosses past vIN varies linearly with the vin value.



You might have noticed the output filter on the class D amplifier on the last slide and though back to our class B and C power amplifiers. Class B amplifiers can be made into push-pulls, but class C amplfiers had a very narrow conduction angle that guaranteed a weird output waveform. Using a resonant output filter can fix that, and that technique is the basis of class C, class E and class F amplfiers. The output filter turns a weird input wave into a sine wave at the vin frequency, and because it's lossless it boosts efficiency when it does so.

The difference between class C and class E amplifiers is that class C amplifiers try to use the conducting device in the linear region for at least a little while, while class E amplifiers are just switched digitally. Class F amplifiers add more output filters to this setup to try to suppress higher harmonics that sneak through the first filter. Each additional filter increases efficiency a bit by squeezing energy back into the vin frequency, but each adds a bit of loss due to parasitics. So there's a careful tradeoff to designing this big filter network.

These filters are popular at RF frequencies where passive components are small, so it's cheaper to build indcutors and capacitors. It's also worth noting that these amplifiers are very efficient, and using a technique called zero voltage switching, where the digital changes are carefully timed with the output waveform, lets class E and class F amplifiers get into the 95% to 99% efficiency range.



Finally, I have to mention one of my favorite curiosities from prehistoric times, the diamond buffer. This is a class AB output stage where the diode drops that cancel the dead zone are provided by a pair of emitter followers. This circuit has very high input impedance and output drive current capacity, and I've never met a problem where it failed to drive a load. It also uses very high voltage rails, which means the biasing diodes on the emitter followers are often made with LEDs. These double as convenient debugging indicators: if one gets too bright then a component is going to start smoking soon. This is a fun one to have in your back pocket.

HARVEY MUDD COLLEGE Summary • There are a lot of fun power amplifiers in the world!