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Miscellaneous Circuit Dynamics

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In this video series we're going to continue finish out our discussion of dynamics by looking at a few specific dynamics examples. The big highlights are that cascode amplifiers are surprisingly fast and that emitter followers have some very odd impedance behaviors, especially if they have large capacitive loads. We're going to talk about differential amplifiers next, and then combine those two topics as we start building our op-amps!

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Cascode Dynamics

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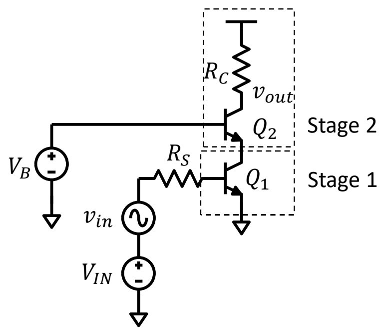
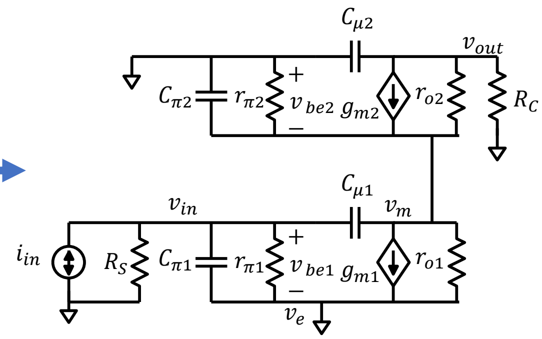
In this video we're going to analyze one dynamic property of cascodes, which will explain why cascode amplifiers often have wide bandwidths.

Cascodes Alleviate the Miller Effect on $C_{\mu 1}$





$r_{out1} = r_{o1}$
$a_{v1} = g_{m1} r_{o1}$
$r_{in2} = 1/g_{m2}$
$a_{v2} = g_{m2} R_C$

$$\frac{v_m}{v_{in}} = a_{v1} \frac{r_{in2}}{r_{out1} + r_{in2}} = -g_{m1} r_{o1} \frac{\frac{1}{g_{m2}}}{r_{o1} + \frac{1}{g_{m2}}} \approx -1$$

$C_{\mu 1}$ is only across a gain of -1, other $OCTC$ fast too b/c $1/g_{m2}$.

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I've included a cascode amplifier on this slide along with a small-signal dynamic model that represents it.

CLICK As a reminder, we analyze cascodes by pretending that they are two stage amplifiers. The first stage is a common emitter, and it is loaded by the second stage, a common base.

CLICK Here are the relevant lab approximations for the amplifier parameters of these stages.

CLICK This is all nice, but it isn't new. However, I wanted to retread this terrain so that I could reveal a neat detail. The gain from the input to the middle node of the cascode, vm, is pretty close to negative 1. That means that Cmu1 isn't aggressively Millerized because the gain across it is fairly small. As a result, cascodes are often much faster than common emitters with the same gain: Cmu gets Millerized by the voltage gain of the common emitter while it does not get Millerized in a cascode.

The other caps in the cascode often have fairly small OCTC too. For instance, Cpi2 sees the 1/gm input impedance of the common base. Cpi1 sees RS||rpi, which is middling, and Cmu2 sees RC||beta*ro, which could be large or small depending on the collector resistance.

We have to calculate the OCTC for each cap because they're all similar and quite small. That's cool, it means cascodes are fast, but it's also unusual. Most amplifiers are dominated by one open circuit time constant. In fact, that behavior is so common and useful that designers will often introduce capacitors into amplifiers to create a big OCTC, a design strategy called dominant pole compensation. More on that later, but if you know an amplifier has a dominant OCTC, then you don't need to bother calculating all the rest of the OCTC.

Summary

- Cascodes are much faster than common emitters because $C_{\mu 1}$ is not Millerized.
- Other caps in the cascode also see low impedances.

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Emitter Follower Dynamics

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In this video we’re going to talk about dynamic behaviors of emitter followers.

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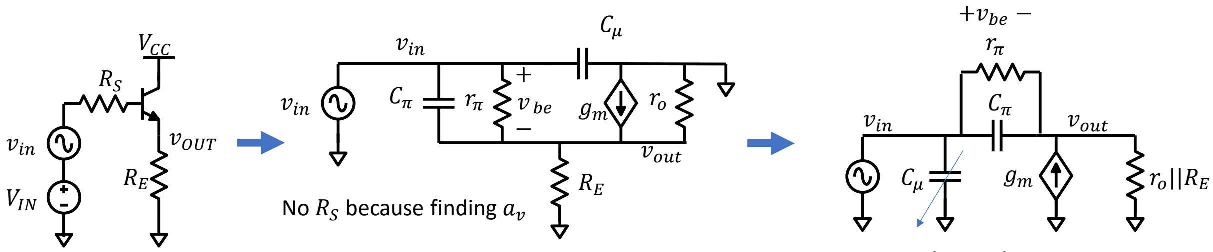
Emitter Follower a_v has a Huge Bandwidth



Ignore if $R_S = 0$

$$z_\pi = r_\pi || \frac{1}{C_\pi s} = \frac{r_\pi}{r_\pi C_\pi s + 1}$$

$$a_v = \frac{(R_E||r_o)(\beta+1)}{(R_E||r_o)(\beta+1) + z_\pi} = \frac{(R_E||r_o)(\beta+1)}{(R_E||r_o)(\beta+1) + r_\pi} \cdot \frac{1 + \frac{r_\pi C_\pi s}{\beta+1}}{1 + \frac{r_\pi C_\pi s}{\beta + 1 + r_\pi/(R_E||r_o)}}$$

Closely spaced pole-zero pair: response flattens out as cap becomes lowest impedance

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I've copied an emitter follower on this page along with two simplifications to it. The second simplification is important because it makes it clear that our dynamic emitter follower is just the same as our DC emitter follower, except that there's a cap in shunt with the whole thing and a cap in parallel with rpi.

CLICK If we combine rpi and Cpi in parallel to make a zpi, then this circuit topology looks exactly like our DC emitter follower with a shunt Cmu in parallel with it. We also get to ignore Cmu when we're finding av because we presume RS is zero when we do that, which shorts out Cmu. That's handy, because it means we can use all of our DC equations for an emitter follower as long as we substitute zpi wherever we would put rpi in our original single-stage equations.

CLICK I've done that for av here, and then simplified the voltage gain into our usual DC gain times a pole zero pair. The pole and the zero are closely spaced in this case. They represent the amplifier's gain going from its DC value all the way up to 1 as Cpi becomes so low impedance that no current flows in rpi, which in turn shuts of the gm generator. This means that decreases in the amplifier's gain mostly come from Cmu interacting with the previous stage's source impedance, and that happens at a very high frequency because Cmu is small.

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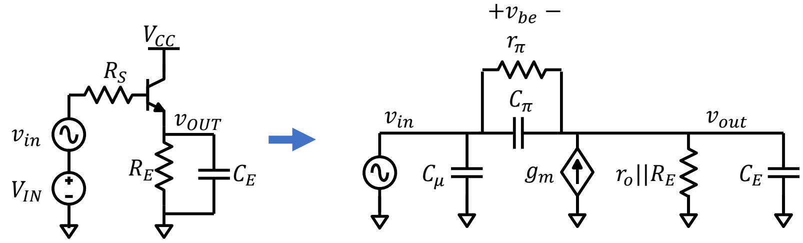
EF w/ Cap Load: z_{in} Can Have –ve Real Part



$$z_\pi = r_\pi || \frac{1}{C_\pi s} = \frac{r_\pi}{r_\pi C_\pi s + 1}$$

$$Z_L = R_E || r_o || \frac{1}{C_E s} \approx \frac{R_E}{R_E C_E s + 1}$$

$$z_{in} = \frac{1}{C_\mu s} || \underbrace{\left[z_{pi} + (1 + g_m z_\pi) Z_L\right]}_{z_{danger}}$$

$$z_{danger} = \frac{r_\pi}{r_\pi C_\pi s + 1} + \frac{R_E}{R_E C_E s + 1} + \frac{g_m r_\pi R_E}{(r_\pi C_\pi s + 1)(R_E C_E s + 1)}$$

$$z_{danger}(j\omega) = \frac{r_\pi}{r_\pi C_\pi j\omega + 1} + \frac{R_E}{R_E C_E j\omega + 1} + \frac{g_m r_\pi R_E}{\underbrace{(1 - r_\pi C_\pi R_E C_E \omega^2)}_{\text{Can be negative! Unstable!}} + (r_\pi C_\pi + R_E C_E) j\omega}$$

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I've copied our emitter follower here because we're going to take a look at its input impedance on this slide. However, I've modified it slightly to add a large emitter cap CE. Load capacitances often make emitter followers unstable, and we'll see why on this slide.

CLICK We're using the same definition of zpi from last slide, and we've also introduced a new compound impedance to represent the load cap in parallel with RE and ro.

CLICK Because we can keep using our emitter follower equations, it's pretty quick to identify that our input impedance is given by Cmu in parallel with a standard emitter follower input impedance where rpi has been replaced by zpi and RE||ro has been replaced by ZL. The emitter follower input impedance can create problems, so I've labeled it as zdanger.

CLICK Expanding out zdanger looks innocent enough, though it's interesting and potentially worrisome that we can get a second order response out of this third term.

CLICK That worry becomes concrete when we substitute j*omega for s to represent a sinusoidal input. We see that there is a term in this expression that could possibly be negative! If that term overwhelms other terms in the expression, then it can result in a net negative real input impedance. Negative real input impedances cause instability, finding one is exactly the same as finding a pole on the right half plane. So beware of big capacitive loads on your emitter followers!

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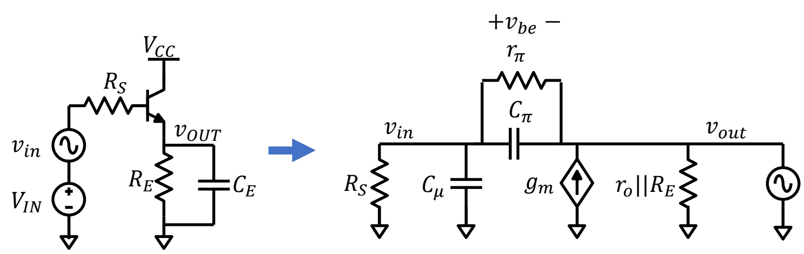
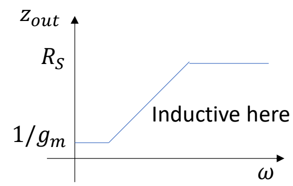
Emitter Follower z_{out} is Pseudo-Inductive



$$z_{out} = R_E||r_o||\frac{z_\pi + R_S||\frac{1}{C_\mu s}}{1 + g_m z_\pi}$$

$$z_{out} \approx \frac{r_\pi + R_S + R_S r_\pi C_\pi s}{1 + r_\pi C_\pi s + g_m r_\pi} = \frac{r_\pi + R_S}{\beta + 1} \cdot \frac{1 + \frac{R_S r_\pi C_\pi s}{r_\pi + R_S}}{1 + \frac{r_\pi C_\pi s}{\beta + 1}}$$



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Finally, it’s worth noting that the output impedance of an emitter follower appears inductive over some frequencies. I’ve copied the emitter follower small signal model here, adding my test source at the output and a source resistance at the input. Note that the source could be any impedance in general, but we’ll confine ourselves to analyzing a resistance for now.

CLICK We calculate the output impedance by substituting zpi into our standard formula

CLICK And then we make a few simplifications. First, we assume the 1/gm term is much smaller than RE or ro. Second, we ignore Cmu for a bit, presuming that it will kick in at high frequency. Thirs, we multiply by the denominator of zpi to start getting this into a manageable form. In the last equality, we afactor out our DC output impedance from the dynamics to zee that the output impedance contains a pole-zero pair. However, the pole is at a much higher frequency than the zero (because the pole is divided by beta+1), so there’s a big range of frequency where the output impedance of the emitter follower is increasing. Inductors increase in impedance with frequency too, so our output impedance looks inductive over a big band. This can result in undesirable ringing, again especially if you have capacitive loads.

Summary

- Emitter followers have wide bandwidth and a high frequency feed-forward zero.
- Emitter followers with large capacitive loads can have negative real input impedances, causing instability.
- Emitter follower outputs look inductive over a wide range of frequencies.

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Current Mirror and Common Base Dynamics

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In this video we're going to look at a few other interesting examples of dynamics. These are useful facts to know if you're implementing high frequency transistor circuits.

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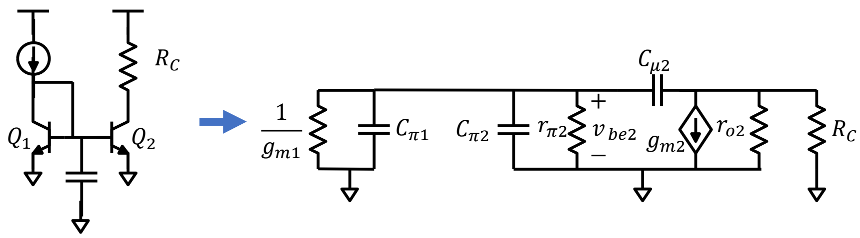
Current Mirrors are Fast, Slow Them w/ Cap



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First, current mirrors are quite fast. That's mostly because Q1 looks like 1/gm, which means Cpi1 and Cpi2 see small impedances. Something similar is going to be true of most mirror topologies because a diode connected device is a crucial part of a mirror.

However, this is kind of bad! Mirrors are biasing circuits, so we don't want them to respond to disturbances. We'd rather they act like low pass filters and basically never change. This can help with stability too.

CLICK So it's common to add a big decoupling capacitor to the middle node of mirrors to slow them down deliberately.

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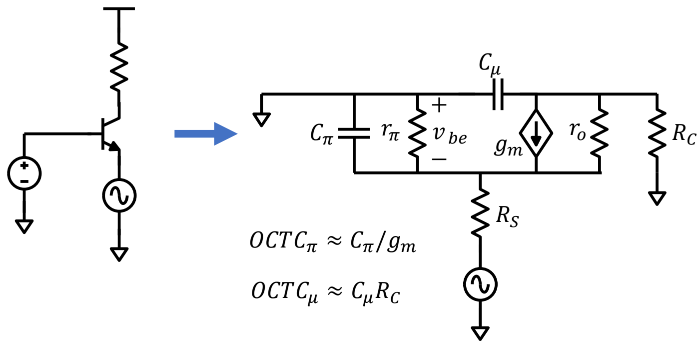
Common Bases are Fast, Have Voltage Gain



$OCTC_\pi \approx C_\pi / g_m$

$OCTC_\mu \approx C_\mu R_C$

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Second, common bases operate at high frequencies and they are pretty well behaved. Both Cpi and Cmu are grounded on one side, so they look like shunt capacitances dangling off of the input and the output. The input impedance of the common base is 1/gm, so the OCTC for Cpi is small. Cmu sees whatever is connected to the collector, but if that can be made small too then this is a very fast amplifier. It also still has voltage gain, which isn't true of the emitter follower. Finally, there's no feedforward zero here because there is no cap directly from the emitter to the collector in our model. Overall, that makes it pretty attractive for high speed applications. Its low input impedance can even be useful there as an interface to 50 ohm RF impedances.

Summary

- Current mirrors have fast dynamic responses, but they're usually bias circuits. Slow them down with caps on the diode connected node.

- Common bases are fast and well-behaved, with very little feed forward behavior.