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Multistage Amplifiers and Emitter Followers

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Up until now we've only looked at common emitter amplifiers, and while they're a fine place to start amplifier desing, they can't do all of the things we'd like amplifiers to do. In this video series we're going to talk about combining amplifiers to achieve specs that we would otherwise be unable to achieve with common emtiters, and we'll also be introducing a new amplifier called the emitter follower. This video series has some important organizational and motivational content for the next few lectures, where we'll be learning about different types of amplifiers that we can use in our multistage designs. We're also going to be analyzing some small signal circuits that turn up over and over again, so you'll need to pay attention both to our process and our results because I want you both to be able to analyze new small signal circuits and to use results that we derive here in bigger designs.

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Multistage Amplifiers

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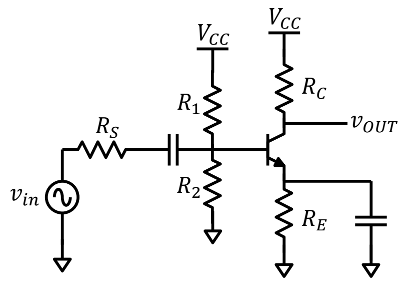
In this video we're going to set our direction for the next few weeks, so it's an important organizational video. We're also going to introduce modeling techniques that let us analyze collections of amplifiers easily.

We Can't Make a CE to Solve Every Problem

- Problem: want 10,000x (80dB) voltage gain and 2V output swing.



$$V_{CC} - V_O = I_C R_C = \frac{V_{SW}}{2}$$ if limited on high side

$$a_v = g_m R_C = \frac{I_C R_C}{\phi_{th}} = \frac{V_{CC} - V_O}{\phi_{th}}$$

For $a_v = 10{,}000$ need $V_{CC} - V_O = 40\text{kV}$

OK, so we're armed with enough amplifier knowledge to be dangerous, let's put it to use. Say a client comes to you requesting an amplifier with a gain of 10,000 and a measly 2V of output swing. This seems like something you can tackle. As a quick aside before we start, some circles will refer to a voltage gain of 10,000 as 80dB of gain, using the formula 20*log(Vout/Vin). I don't love that definition because of some subtleties about the source and load impedance, but don't forget dBs, they come up often when you're designing around large numbers in amplifiers.

CLICK OK, so let's get into this design. We know that the large signal voltage drop across RC is given by IC*RC, and that sets VO. You should note that if VO is closer to VOMAX than VOMIN, this value is also VSW/2.

CLICK However, you might note with dismay that our voltage gain is equal to gm*RC, and if we sub in for gm then we find the gain is IC*RC/phi_th, and substituting in the previous line reveals that our voltage gain is directly proportional to the voltage drop across RC. This relation is true regardless of what IC and RC you pick, so we know that every volt of drop across RC turns into a voltage gain of about 40. (because 1/25mV is about 40 1/V)

CLICK The consequences of that become clear if we calculate how much dropout we'd need to achieve a gain of 10,000. Rearranging the equation above shows that VCC-VO would have to be 40kV in order to get a gain of 10,000. That's implausible for a wide variety of reasons, so it looks like the common emitter can't be used to make this amplifier design work.

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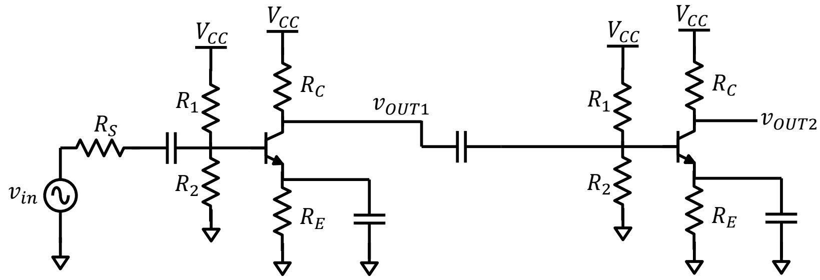
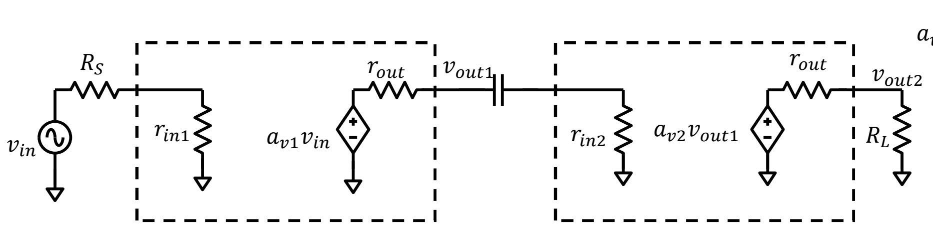
Combining Amp Stages Makes Super Amps



$a_v = 200$ implies $V_{CC} - V_O = 5\text{V}$

Two stages:
$a_v \approx 20{,}000$ with 10V swing
(depending on r_{out} and r_{in})

Meets our design spec!



$$a_{v,multi} = a_{v1}\frac{r_{in2}}{r_{out1} + r_{in2}}a_{v2}$$

$$r_{in,multi} = r_{in,1}$$

$$r_{out,multi} = r_{out,2}$$

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OK, that's rough, but what if we stuck two common emitter amplifiers together? Maybe this would produce the gain we needed without needing a crazy VCC value. It certainly looks like a nice idea, but the idea of re-analyzing some big structure with two small signal models inside of it seems like a pain. Notice that I've coupled the amplifiers together with a capacitor, so they can each have different DC bias points.

CLICK Fortunately, we can use our generic amplifier model to make our lives much easier. If we know rin, rout and av for each amplifer then we can convert them to this generic form, and it's really easy to calculate the gains and impedances associated with this multistage amplifier.

CLICK The gain of the multistage amplifier is given by the product of the gain of each amplifier multiplied by the voltage dividers formed between each stage, called interstage loading. When this multistage amplifier is put in a circuit, it will see extra dividers at the load and the source as usual. The output impedance of the combined amplifier is given by the output impedance of the last stage, and the input impedance is given by the input impedance of the first stage.

CLICK Taking that back to our two common emitters, we can design each with av of 200, which means we'll have a 5V drop from VCC on each of them.

CLICK That means if the rout of the first stage is similar to the rin of the second stage, we'll pick up a gain of ~20,000 from these two amplifiers with around 10V of swing. Nice!

This is called a multistage model

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Heterogeneous Multistage Allows Even More

Issue with CE Amp	Amplifier to Fix It
r_{out} often high and r_{in} is modest	Emitter Follower
Poor linearity / gain depends on g_m	CE with Degeneration Resistor
Gain and swing are coupled	CE w/ Degeneration, Active Load
a_v isn't super high	Cascodes with active loads

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We can go even further with multistage amplifiers by replacing some of the common emitter stages with other amplifiers. On this slide I've listed issues with common emitters and the amplifiers that we're going to use instead of common emitters to fix them. For instance, we can get better rout and rin from emitter follower amplifiers than we can with common emitters, and we can fix that lingering gm non-linearity for large gains by using something called a common emitter with degeneration resistance. There are other problems we'll address too: we'll look at a few ways to fix the coupling of gain and swing, including the aforementioned common emitter with degeneration and also something called an active load, and we'll find something called a cascode can make comically high voltage gains.

All of this is hopefully interesting foreshadowing. But the main takeaway is that we're going to study some other amplifiers that we can include in our multistage amplifier model in the next few lectures.

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Summary

- Common emitter amplifiers can't do everything, partially because of a fixed relationship between gain and swing:

$$a_v = \frac{V_{CC} - V_O}{\phi_{th}}$$

- Using multistage amplifiers lets us combine properties from many amplifiers:

$$r_{in,multi} = r_{in,1} \qquad a_{v,multi} = a_{v1}\frac{r_{in2}}{r_{out1} + r_{in2}}a_{v2} \qquad r_{out,multi} = r_{out,2}$$

- We're learning about new amplifiers with new properties next.

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Emitter Followers

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In this video we're going to find the small signal parameters of an amplifier called the emitter follower. This might sound simple, and that's true in the sense that each step in the following slides is pretty easy, but this video is going to cover a lot of ground. That's particularly true because I want you to remember both how we analyzed the emitter follower small signal model and the results of the analysis. There are only two videos today, so it's important that you take this one slowly and absorb it. Also pause and work problems when requested.

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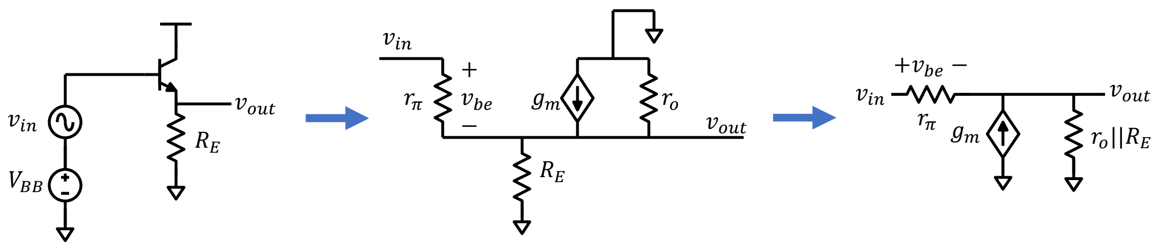
Emitter Follower: Emitter Not Grounded

- AKA Common Collector b/c collector is grounded.
- Will find they have high r_{in}, low r_{out}, $a_v \approx 1$.
- Good as voltage buffers, reduce loading effects.



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I've drawn an emitter follower on the left of this slide and listed a few of it's notable or desirable properties on the left. A handful of people call the emitter follower a common collector amplifier because the collector is small signal grounded by being connected to VCC. Importantly, that means the emitter is not grounded, which makes calculating input and output resistances much more complex. When we calculate the amplifier parameters we'll find that rin of an emitter follower is very high, rout is very low and av is about one. That makes emitter followers great voltage buffers: they can make copies of voltages and drive them with low output impedance.

We need to draw a small signal model of the emitter follower to start calculating those amplifier parameters. I'd like you to pause the video and draw a small signal model for an emitter follower.

CLICK Here's my rendition. Note that the collector is grounded, the emitter is now free to move around, which is important because it's the output node of this amplifier.

CLICK We can clean this schematic up considerably by getting rid of some of the extra 90 degree angles and combining ro and RE in parallel. We'll use this model to find rin, rout and av.

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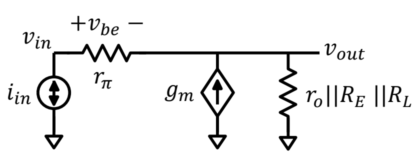
Emitter Follower r_{in}

- Set $R_S = 0$, R_L proper value
- R from v_{out} to ground is $R_E || r_o || R_L$
- Use i_{in} as test source: easier than v_{in}



$$v_{be} = i_{in} r_\pi$$

$$v_{out} = (i_{in} + i_{gm})(R_E || r_o || R_L)$$
$$= i_{in}(1 + g_m r_\pi)(R_E || r_o || R_L)$$
$$v_{in} = v_{out} + v_{be}$$
$$= i_{in}(1 + g_m r_\pi)(R_E || r_o || R_L) + i_{in} r_\pi$$

$$r_{in} = r_\pi + (\beta + 1)(r_o || R_E || R_L) \approx \beta R_E$$

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We set up our analysis to find rin by remembering the test conditions we use for rin. We set RS to zero so it doesn't add to the rin that we calculate, and we set RL to an appropriate value so that any feedback effects in our amplifier get captured by rin. Because RL would be connected between vout and ground, it just falls in parallel with our ro||RE resistor from the previous page.

Because this model is more complex than the common emitter, we can find the Thevenin resistance that represents rin just by calculating equivalent resistances. The dependent source means we need to apply a test source. We've opted to use an input test current to find rin because it happens to be easier. Remember that you have the freedom to use either input test currents or input test voltages.

Given this setup, I'd like you to pause the video and try to find rin. You might thrash with the algebra a bit, and that's perfectly normal when you're getting used to small signal model analysis.

CLICK First we notice that our gm control variable, vbe, is equal to iin*rpi. That's super handy because it means we can sub in expressions for our dependent source current in terms of our input. Finding your dependent source control variable as soon as possible is always a good idea.

CLICK We want to calculate the output node voltage because we can add it to vbe to find vin. It's given by the sum of the gm current and the rpi current being driven through the resistor to ground.

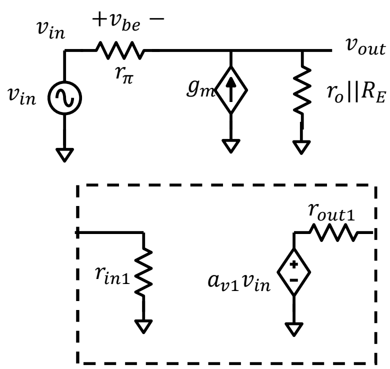
CLICK We know the gm current is given by gm*vbe, and vbe is given by iin*rpi, so substituting both of those relationships in here and factoring out iin gives us this expression.

CLICK Great, we know vin is equal to vout+vbe,
CLICK and we have expressions for each of those voltages we can substitute here.
CLICK and we finally divide iin off the right side to find that rin is equal to rpi + (beta+1)(ro||RE||RL). That's a very good expression for rin, but if you're in lab and don't want to memorize the whole thing then you can use the very quick and dirty approximation that rin is about beta*RE. This is a great expression for doing math in your head, which I recommend, but you'll want to use the more exact expression for matching to simulations and measurements.

Here's an expression and a super-abbreviated expression …

Emitter Follower a_v

- Set $R_S = 0$, $R_L = \infty$
- Must use test voltage



$$v_{out} = (i_{in} + i_{gm})(R_E||r_o)$$

$$= i_{in}(1+\beta)(R_E||r_o)$$

$$= \frac{v_{in}}{r_{in}}(1+\beta)(R_E||r_o)$$

$$a_v = \frac{(\beta+1)(R_E||r_o)}{r_\pi + (\beta+1)(R_E||r_o)} \approx 1$$

This makes sense because $v_E = v_B - V_{BEON}$

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Now we're going to find the voltage gain of the emitter follower. We recall the we calculate voltage gain as small signal vout over small signal vin with zero source resistance and infinite load resistance. We set the resistances that way so that we're not measuring either the source or load divider by accident. I've set that up in the small signal model below, note that the resistor from vout to ground has gone back to ro||RE because we removed the load. Pause the video and try to find the gain of this amplifier.

CLICK OK, we can start by finding an expression for vout. This one ought to look familiar from last slide: we know the current through rpi and the current through gm have to go to ground through the ro||RE resistance, which creates the output voltage.

CLICK A little factoring and recalling that vbe is equal to iin*rpi and beta is gm*rpi gets us to this expression. However, we've got a problem, we don't know iin like we did when we had a test current source.

CLICK So we're going to play a cheeky trick. We know that this amplifier can be effectively represented by its small signal model, and that rin describes the relationship between vin applied to the base and current into the base.

CLICK So we can just say that iin is equal to vin/rin from the previous slide. This is fun, we've jumped back and forth between our amplifier model and our transistor model to figure out the current into the base. However, I've also seen students get tripped up because this trick only works because of our silly biasing. We needed the current through rpi (into the base) to plug into this second expression, and I've seen people get confused and use a current through a bias network in this step instead. So this is another reminder that we find our amplifier parameters assuming silly voltage source biasing, and you need to modify these expressions to account for bias networks.

CLICK Subbing in our expression from the previous page, we find that the voltage gain is given by (beta+1)*RE||ro / (beta+1)RE||ro + rpi. If beta*RE is bigger than rpi, then this gain is quite close to one, which is the quick and dirty in the lab approximation for emitter follower gain.

CLICK The gain of one makes particular sense with emitter followers because the total signal relation between input and output voltages in an emitter follower is that vE is just a VBEON drop below vB. If we assume VBEON is constant, then plotting vE vs. vB has a slope of one. If the total signal relationship has a gain of about one, the small signal relationship has to have that behavior too.

Careful with this trick! This is rin looking into a base

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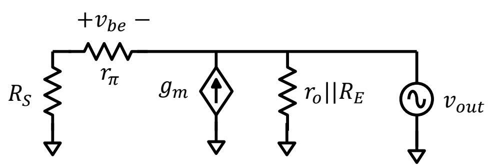
Emitter Follower r_{out}

- Set $R_L = \infty$, R_S to proper value
- Test v_{out} happens to be easier than i_{out}



$$i_{out} = \frac{v_{out}}{r_o} + \frac{v_{out}}{R_E} + g_m \frac{r_\pi}{R_S + r_\pi} v_{out} + \frac{v_{out}}{R_\pi + R_S}$$

$$r_{out} = \frac{1}{\frac{1}{r_o} + \frac{1}{R_E} + \frac{g_m r_\pi + 1}{R_S + r_\pi}}$$

$$r_{out} = r_o || R_E || \left(\frac{R_S + r_\pi}{\beta + 1}\right) \approx \frac{1}{g_m} + \frac{R_S}{\beta}$$

$1/g_m$ looks like a resistance because we directly control v_{be}.

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Finally, we're going to find rout of the emitter follower. Our rout test conditions are an infinite load resistance, so it doesn't steal any current from our output, and an RS that is normal for the amplifier. We're choosing to use a test voltage source for this analysis because it makes our life easier. Pause the video and try to find rout.

CLICK We start by writing that iout is the sum of current in a bunch of parallel branches: resistive current in the ro, RE and RS+rpi branches, and a gm current that is controlled by a voltage divider between rpi and RS. We played a few tricks here. First we wanted the voltage on the top resistor in the voltage divider, which means rpi is on top. Second, vbe is activated in a negative direction when vout increases, but that also causes the gm source to inject current to enter the vout node. We can switch the signs on vbe and the direction of the arrow on gm and wind up with the same result. This has the effect of just cancelling a few negative signs in the gm term.

CLICK We factor out vout and divide through, then invert the equation to find rout. Note that I merged the two terms with RS+rpi in their denominators. Not also that rout looks like the inverse of a bunch of inverse resistances.

CLICK So that means we can rewrite rout as a parallel combination of ro, RE and this weird pseudo-resistance that is RS+rpi / (beta+1). We can make a lab-worthy approximation for this of 1/gm + RS/beta, by assuming the pseudo-resistance term is the smallest in our equation, saying beta is pretty close to beta plus one, splitting up the numerator, and remembering beta is equal to gm*rpi. This is often shortened further to rout being equal to 1/gm, but don't forget about the effect of RS/beta if your source resistance is big or your rout constaint is tight.

CLICK One fun note is that 1/gm is showing up here as a resistance. At first glance, that's cute because the units are right. At second glance, it makes even more sense: we're moving vout up and down, which directly affects the control variable for gm. In response, gm is injecting a linear amount current into the out node. An resistor is an element that has a linear current-voltage relationship to changes in one node, so the arrangement of the control variable here makes gm behave like a resistor. That means any time we wiggle an emitter around, we should expect to see an impedance that looks something like 1/gm.

That last fact implies that we'll wiggle emitters around in other circuits, which is true. As a result, it can help to remember how we solved this particular circuit to re-apply it in the future. The same is true of the emitter follower input resistance. I call these common dependent source and resistor arrangements small signal patterns, and we're going to meditate on them for a bit in future videos.

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Summary

- Emitter followers are good voltage buffers: high r_{in}, low r_{out}, $a_v \approx 1$

- Emitter follower amplifier parameters

$$r_{in} = r_\pi + (\beta + 1)(r_o || R_E \, || R_L) \approx \beta R_E$$

$$r_{out} = r_o || R_E \, || \left(\frac{1}{g_m} + \frac{R_S}{\beta} \right) \approx \frac{1}{g_m} + \frac{R_S}{\beta}$$

$$a_v = \frac{(\beta + 1)(R_E || r_o)}{r_\pi + (\beta + 1)(R_E || r_o)} \approx 1$$

- Saw two small-signal circuit solving patterns that will show up again.

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