E151 Lecture 18 – Open and Short Circuit Time Constants

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Disclaimer

These are notes for Prof. Spencer to give the lecture, they were not intended as a reference for students. Students asked for them anyway, so I'm putting them up as a courtesy. Remember that they are not intended as a substitute for attending lecture.

Lab Debrief

- AC coupling
  - We're not modeling it in our TF right now, low f will be wrong
  - step responses: see dynamics right close to edge
- Extracting Cmu and Cpi
  - assume Cce=0, Cobo=Cmu, use fT to get Cpi
  - OR, use Cibo + tau_F and gm
- Some weirdness in this lab measuring rin and rout … play around a bit
- Make sure to record phase for Bode plots
- <u>MAKEUP LAB HOURS</u> on Wednesday, 6-9PM

Full TF Hard (no good Zth) & Miller has issues

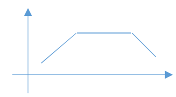
- Miller issues: CE specific and non-conservative, but insightful

- We Want
  - a general approximation method that works for any circuit
  - approximation to be conservative & to lend design insight
  - a technique that can be used to calculate f_low

- Open circuit time constants & short circuit time constants (O/SCTC)
  - Invented at MIT in the 60s

What are OCTC?

- Assume our transfer function has a mid-band



- Focus on finding first pole accurately (assume widely separated)
- “zero-causing caps” are going to be shorted at this frequency

Real CE: $H(s) = A_v \frac{(1 - s/z)}{(1-s/p_1)(1-s/p_2)}$, Miller: $H(s) = A_v \frac{1}{1 - s/p_M}$ ← lose info, use R_{TH} of C_M to get p_M

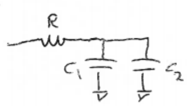
OCTC: $H(s) = A_v \frac{1}{(1-s/p_1)(1-s/p_2)\cdots(1-s/p_n)} = A_v \frac{1}{1 + \underbrace{\left(\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}\cdots\frac{1}{p_n}\right)}_{\text{same as } \tau_1+\tau_2+\cdots} s + \cdots + \underbrace{\left(\frac{1}{p_1 p_2 \cdots p_n}\right)}_{\text{same as } \tau_1\tau_2\cdots\tau_n} s^n}$

$H(s) \approx A_v \frac{1}{1+(\tau_1+\tau_2+\cdots+\tau_n)s}$ Because tau_n*s small @ corner for n>2

Fine, but I don’t know τ_i

- Can’t Thevenize @ caps b/c the caps communicate in general
- **Big result**: $\sum_{i=0}^{n} \tau_i = \sum_{i=0}^{n} OCTC_i$ where $OCTC_i = C_i R_{th,i} | C_{k \neq i}\ open$
- Exactly predicts first pole in OCTC TF
- Only include “pole-causing” caps in analysis b/c “zero causing” short

Example (trivial)



Exact

$\tau = R(C_1+C_2)$

OCTC

$OCTC_1 = C_1 R$

$OCTC_2 = C_2 R$

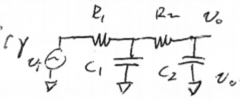
$\sum OCTC = R(C_1+C_2)$

- Expect 1st order to work perfectly based on TF

Less Trivial Example

Lest trivial example

You Try



Exact

$$v_o = \frac{\frac{1}{C_2 s} \cdot v_{th}}{\frac{1}{C_2 s} + Z_{TH}}$$

$$Z_{TH} = R_2 + R_1 \| \frac{1}{C_1 s} = R_2 + \frac{R_1}{1+R_1C_1 s}$$

$$v_{TH} = \frac{1/C_1 s}{R_1 + 1/C_1 s} v_i = \frac{v_i}{1+R_1C_1 s}$$

vs. OCTC

$OCTC_1 = C_1R_1$

$OCTC_2 = C_2(R_1+R_2)$

3_{dB} @ $\left(R_1C_1 + (R_1+R_2)C_2\right)^{-1}$

$$\frac{v_o}{v_i} = \frac{\frac{1}{C_2 s}\cdot\frac{1}{1+R_1C_1 s}}{\frac{1}{C_2 s} + R_2 + \frac{R_1}{1+R_1C_1 s}} = \frac{1}{(1+R_1C_1 s) + R_2C_2 s(1+R_1C_1 s) + R_1C_2 s} = \frac{1}{1 + (R_1+R_2)C_2 s + R_1R_2C_1C_2 s^2 \dots + R_1C_1 s}$$

$\hookrightarrow (1+s/p_1)(1+s/p_2)$

- Works for p1! Esp. good if tau_1>tau_2 → Miller helps

Failure Mode if Poles are Closely Spaced

– what happens if poles are close?

$$\frac{1}{1+(\tau_1+\tau_2)s + \tau_1\tau_2 s} \xrightarrow{FRF} \frac{1}{1+\underbrace{(\tau_1+\tau_2)\omega_{-3dB,est}}_{} + \underbrace{\tau_1\tau_2\,\omega_{-3dB est}}_{}}$$

= 1 by def^n of OCTC

each of $\tau_1 + \tau_2$ must be <1, so product much smaller

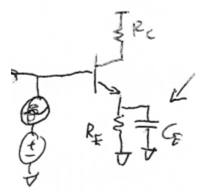
worst case, $\tau_1=\tau_2 \rightarrow$ this is $\frac{1}{4}(\tau_1+\tau_2)\omega^2$

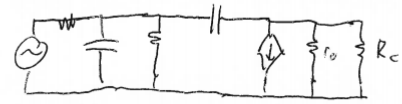
– Also, even if this doesn't give us the exact ω_{-3dB}, it does give us → ① a good guess

→ ② design insight about which caps cause trouble

Try a CE + a pattern for Cmu

- Ignore Ce and coupling caps b/c zero-causing, not pole-causing



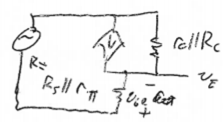


You guys find $OCTC_{\pi}$

$= (R_s \| r_\pi) C_\pi$

- Cmu is more complicated:

$OCTC_{\mu}$



Compare

Exact

$\tau_1 = C_\pi R + C_\mu (R + R_c + g_m R R_c)$

Miller

$\tau_1 = C_\pi R + C_\mu (R + g_m R R_c)$

we beat miller!

SCTC

OCTC reminder

Let $H(s) = \dfrac{A_v}{(1+\tau_1 s)(1+\tau_2 s)\cdots} = \dfrac{1}{1+\sum \tau + \cdots}$

$= \dfrac{A_v}{1 + s\sum \tau_i + \cdots + \prod \tau_i \cdot s^n}$

Assert low order terms matter b/c @ high f close to τ_i

SCTC

Let $H(s) = \dfrac{A_v k s^n}{(s+p_1)(s+p_2)(s+p_3)\cdots} = \dfrac{A k s^n}{s^n + s^{n-1}\sum p + \cdots + \prod p_i}$

~ Alternate form of poles, need k

~ High pass - like behavior

Assert high order terms matter b/c @ $\dfrac{\prod p_i}{s^{n-1}}$ small at low f near corner

so $H(s) = \dfrac{A k s}{s + \sum p_i}$

- Not associated w/ particular zero
- Pole-causing caps open
- Find f_low
- Find 2nd order pole w/ OCTC

→ pole located @ sum of pole frequencies, still hard to find

$\sum \tau_i = \sum OCTC_i$ where $OCTC_i = C_i \cdot R_{TH}\big|_{\text{caps open}}$

$\sum p_i = \sum SCTC_i^{-1}$ where $SCTC_i = C_i \cdot R_{TH}\big|_{\text{All other caps short}}$