E151 Lecture 17 – CE Dynamics and Miller

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Disclaimer

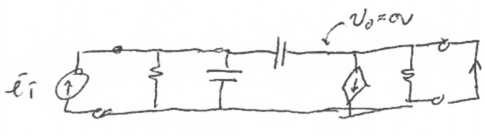
These are notes for Prof. Spencer to give the lecture, they were not intended as a reference for students. Students asked for them anyway, so I'm putting them up as a courtesy. Remember that they are not intended as a substitute for attending lecture.

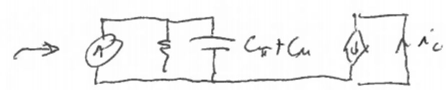
Describe BJT Small Signal Speed Limit with fT

How fast can we go?

– often measured w/ f_T, frequency where A_i drops to 1

f_{max} where power gain = 1





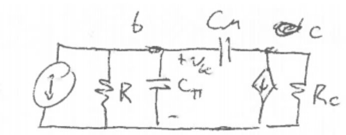
$i_o = g_m v_{be}$

$v_{be} = i_i \cdot \frac{R/(C_\pi + C_\mu)s}{R + 1/(C_\pi + C_\mu)s} = \frac{r_\pi}{1 + r_\pi (C_\pi + C_\mu)s}$

So $A_i = \frac{\beta}{1 + \beta\left(\frac{C_\pi + C_\mu}{g_m}\right)s}$

$\to f_T \approx \frac{1}{2\pi} \frac{g_m}{C_\pi + C_\mu}$

Dynamic Model of CE Amplifier



$R = R_S \,||\, r_\pi$

$i_i = v_i / R_S$

Let R_C stand for $R_C || r_o$

KCL @ b

$i_i = \frac{v_b}{R} + v_b C_\pi s + (v_b - v_c) C_\mu s$

KCL @ c

$(v_b - v_c) C_\mu s = g_m v_b + \frac{v_c}{R_C}$

$\longrightarrow \frac{v_o}{v_i} = -g_m R_C \frac{r_\pi}{R_S + r_\pi} \cdot \frac{1 - s\frac{C_\mu}{g_m}}{1 + s\left(C_\mu(R_L + R_S||r_\pi + g_m R_L R_S||r_\pi) + C_\pi R_S||r_\pi\right) + s^2 C_\mu C_\pi R_L R_S||r_\pi}$

Terrible Algebra Arrow!

- Separates into gain + dynamics, get gain from DC circuit
- RHP zero → negative feedthrough
- This is unsustainable … need approximation methods

For Clarity: $\frac{\left(1 - \frac{sC_\mu}{g_m}\right)}{1 + s\left(C_\mu\left(R_L + R_{s||\pi} + g_m R_L R_{s||\pi}\right) + C_\pi R_{s||\pi}\right) + s^2 C_\mu C_\pi R_L R_{s||\pi}}$

First Simplification: CE Amplifier PZ Plot

- We can simplify our life by noticing poles are real & widely separated

↳ poles usually real or widely separated

let $D(s) = (1+\tau_1 s)(1+\tau_2 s) = 1 + (\tau_1+\tau_2)s + \tau_1\tau_2 s^2 \approx 1 + \tau_1 s + \tau_1\tau_2 s^2$

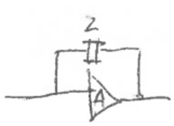
dominant pole is slow

$$\begin{cases} P_1 = \frac{1}{\tau_1} = C_\pi R_s \| r_\pi + C_\mu(R_L + R_s \| r_\pi + g_m R_L R_s \| r_\pi) \\ P_2 = \frac{1}{\tau_2} = \frac{1}{R_E C_\mu} + \frac{1}{R C_\pi} + \frac{1}{R_L C_\pi} + \frac{g_m}{C_\pi} \end{cases}$$

- Communicating caps … icky … can't just thevenize
- Assumed tau1 >> tau2 so we can pull tau1 straight from s coefficient
  - True because of gm*Rl term
  - Why is our voltage gain in here?

The Miller Effect

Thevenize here

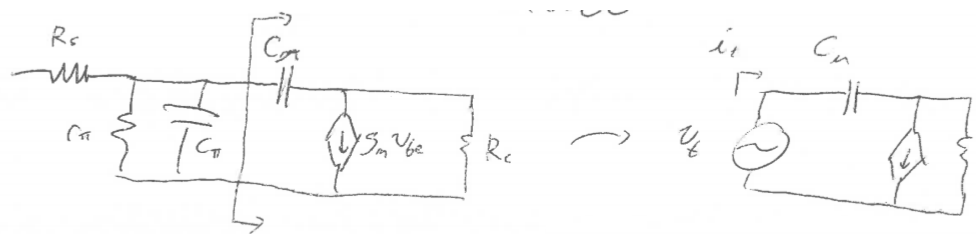


$$i = \frac{v_i - v_o}{Z} = \frac{v_i}{Z} \cdot (1-A)$$

$$\frac{v}{i} = Z_{eff} = \frac{Z}{1-A}$$

- Thevenin impedance is boosted by gain across a feedback element!
- Caps in feedback around negative gain appear bigger by 1+A

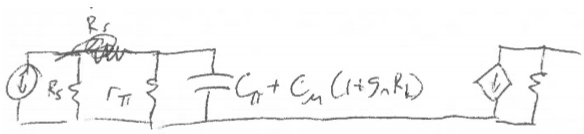
The Miller Approximation for a CE



Find thevenin here

$$\frac{v_t - v_o}{1/C_\mu s} = i_t = C_\mu s(v_t - v_o)$$

- Let’s just calculate the first pole & assume that Cmu sees the DC gain



$$\tau_1 = (R_s \| r_\pi)(C_\pi + C_\mu(1 + g_m R_L))$$

$$v_o = -g_m R_c \frac{R_s \| r_\pi}{1 + \tau_1 s} \cdot \frac{v_i}{R_s}$$

Issues with Miller Approximation

- Not conservative:
  - lose zero and 2nd pole, so ignore lots of stability implications
  - Approximation of tau_1 loses a Cmu*Rc term, appears a bit faster
- Kind of CE specific,
  - relies on av falling across tricky feedback cap
  - lets us convert C_mu to grounded
- Even so, gives good intuition about caps in feedback!