E151 Lecture 15 – References

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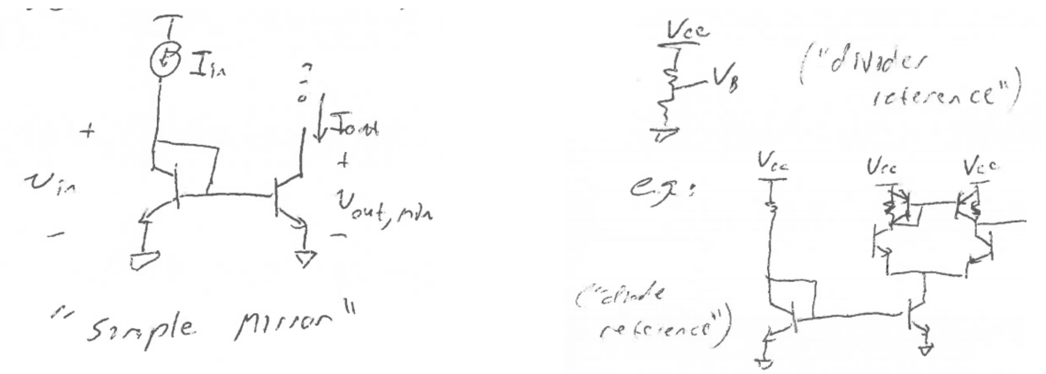
Disclaimer

These are notes for Prof. Spencer to give the lecture, they were not intended as a reference for students. Students asked for them anyway, so I'm putting them up as a courtesy. Remember that they are not intended as a substitute for attending lecture.

How Do We Make Current Sources in Mirrors?



- Need analysis tools to measure "how good" V or I reference is
- References need to be supply and temperature invariant (PVT is big)

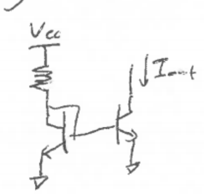
Measure Quality of Reference with Sensitivity

called sensitivity

$$S^{\text{of } Y(x)}_{\text{to } x} \equiv S^{Y}_{x} \equiv \lim_{\Delta x \to 0} \frac{\Delta Y / Y}{\Delta x / x} = \frac{x}{Y}\frac{\partial Y}{\partial x}$$

fractional change of Y w/rt X

e.g.: diode reference in simple mirror



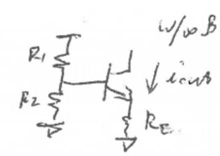
$$S^{I_{out}}_{V_{cc}} = \frac{V_{cc}}{I_{out}} \cdot \frac{\partial}{\partial V_{cc}}\left[\left(\frac{\beta}{\beta+2}\right)\cdot\left(\frac{V_{cc}-V_{be,on}}{R}\right)\right]$$

$$= \frac{\frac{\beta}{\beta+2}\cdot\frac{V_{cc}}{R}}{\left(\frac{\beta}{\beta+2}\right)\left(\frac{V_{cc}-V_{be,on}}{R}\right)}$$

$$\approx 1 \Big/ \left(1 - \frac{V_{be,on}}{V_{cc}}\right) \approx 1$$

very sensitive!

you try



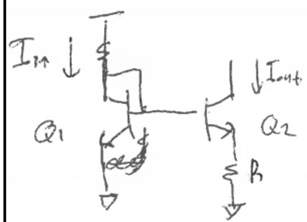
$S^{I_{out}}_{V_{cc}}$:

$$I_{out} = \left(\frac{R_2}{R_1+R_2}V_{cc} - V_{be,on}\right)\cdot\frac{1}{R_E}$$

$$\frac{\partial I_{out}}{\partial V_{cc}} = \frac{R_2}{R_E}\cdot\frac{1}{R_1+R_2}$$

$$S^{I_{out}}_{V_{cc}} = \frac{1}{\left(1-\left(1+\frac{R_1}{R_2}\right)\frac{V_{be,on}}{V_{cc}}\right)} \approx 1$$

“Widlar Current Source” Reference if Time



Interested in $I_{out}(I_{in})$

- $v_{be1} = v_{be2} + I_{out} R \cdot \frac{\beta}{\beta+1}$
- $I_{s1} e^{v_{be1}/\phi_T} = i_{C1} \rightarrow v_{be1} = \phi_T \ln \frac{i_{C1}}{I_{s1}}$
- $\phi_T \ln \frac{I_{in}}{I_{out}} \approx I_{out} R$

• Note can't assume 0.7V here

• Need an $1/\beta$ for v_{be} (bugs last lecture)

- Logarithmic compression of Iin, so called a reference

“Widlar Current Source” Reference if Time

- Sensitivity w/ 1mA and 1k this is a factor of 40 better

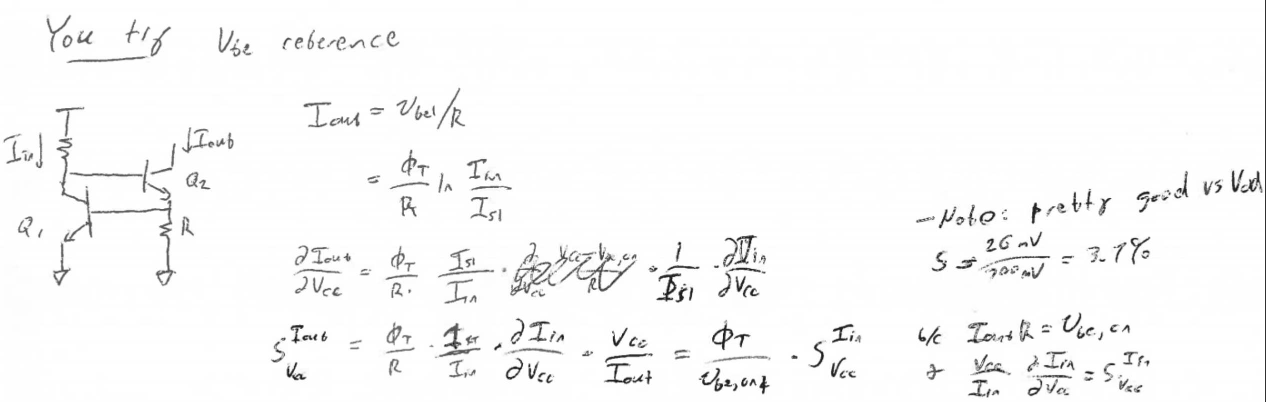
Sensitivity ~ $\phi_T \frac{\partial}{\partial V_{cc}} \ln \frac{I_{in}}{I_{out}} = R \frac{\partial I_{out}}{\partial V_{cc}}$

$$\phi_T \left(\frac{1}{I_{out}} \frac{\partial I_{in}}{\partial V_{cc}} - \frac{I_{in}}{I_{out}^2} \frac{\partial I_{out}}{\partial V_{cc}} \right) = R \frac{\partial I_{out}}{\partial V_{cc}}$$

$$\frac{\partial I_{out}}{\partial V_{cc}} = \left(\frac{1}{1 + \frac{I_{out} R}{\phi_T}} \right) \cdot \frac{I_{out}}{I_{in}} \cdot \frac{\partial I_{in}}{\partial V_{cc}}$$

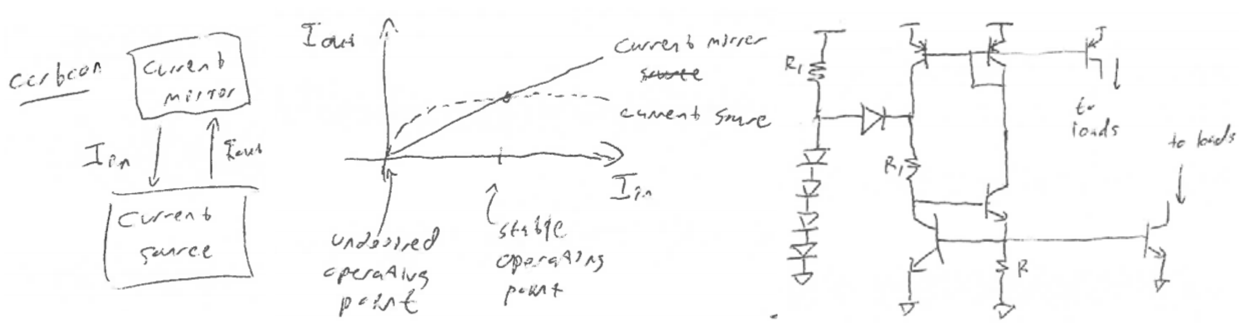
$$S^{I_{out}}_{V_{cc}} = \frac{V_{cc}}{I_{out}} \cdot \frac{\partial I_{out}}{\partial V_{cc}} = \left(\frac{1}{1 + I_{out} R / \phi_T} \right) \cdot \underbrace{S^{I_{in}}_{V_{cc}}}_{\approx 1} \approx \left(\frac{1}{1 + \frac{I_{out} R}{\phi_T}} \right)$$

Vbe Reference if Time



- I like this one, easy to implement, easy to analyze, good vs. supply

Self-Biasing for Supply Insensitivity



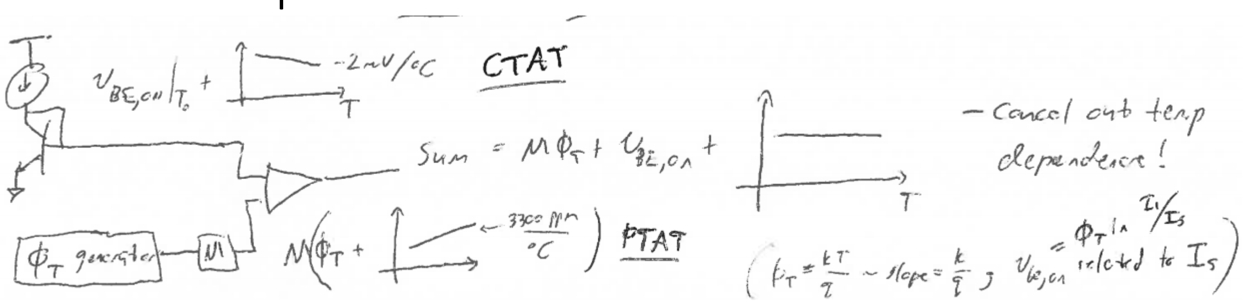
- Get away from dropping Vcc over R … big sensitivity benefit
- Need startup circuit because undesired stable point (eg: 4Vbe vs. 3)
- Want startup circuits to shut themselves off (eg: rev. bias b/c of Rx)

Temperature Sensitivity

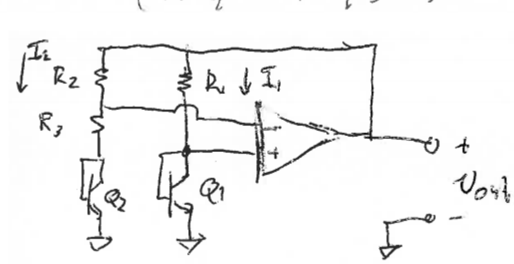
$$TC^{I_{out}} = \frac{1}{I_{out}} \frac{\partial I_{out}}{\partial T}$$

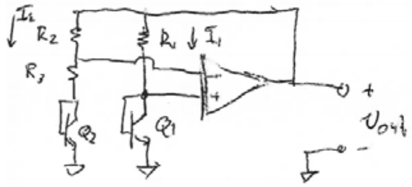
- Model w/ temperature coefficient (sensitivity to temp, % change/deg)
- Big issues: Resistors (+1100 ppm/C), Vbe (-2mV/C), ϕ_T (3300 ppm/C)
- Cancel OK in Widlar, bad issue in Vbe, assorted self-bias tricks
- Sometimes need reference V constant w/ both supply and temp (eg: regulators) → Band Gap

Band Gap Reference



- Very literal implementation →





- op-amp forces $v_{R1} = v_{R2}$
- so $\frac{I_1}{I_2} = \frac{R_2}{R_1}$
- ϕ_T generating: can get it from Δv_{be}

• $v_{R3} = \underbrace{v_{be1} - v_{be2}}_{\Delta v_{be}} = \phi_T \ln \frac{I_1}{I_{s1}} - \phi_T \ln \frac{I_2}{I_{s2}} = \phi_T \ln \frac{R_2}{R_1} \frac{I_{s2}}{I_{s1}}$

• same current in R_2 & R_3, so $v_{R2} = \frac{R_2}{R_3} v_{R3}$

Finally, do KVL, $v_{out} = v_{be2} + v_{R2} + v_{R3}$

$$= v_{be2} + \left(1 + \frac{R_2}{R_3}\right) \phi_T \ln \frac{R_2}{R_1} \frac{I_{s2}}{I_{s1}}$$

$$= v_{be2} + M\phi_T \quad \text{where} \quad M = \left(1 + \frac{R_2}{R_3}\right) \ln \frac{R_2}{R_1} \frac{I_{s2}}{I_{s1}}$$

Band Gap Details

- V_OUT = 1.26V, about the band gap of silicon in eV (but unrelated)
- Temperature variation: parabolic, concave down, a few mV over 100C
- Implement in CMOS w/ parasitic susbstrate PNP