Small-Signal Tricks

**Flipping Direction**  Consider the following simple small-signal model:

![Small-Signal Tricks Diagram](image)

Sometimes, it’s convenient to flip the direction of the voltage-controlled current source (for instance, to match one of the other tricks in this handout). In this case, $v_{be}$ must be flipped too, either on $r_\pi$ or in the voltage-controlled current source’s equation:

![Small-Signal Tricks Diagram](image)

**Effective $g_m$**  Compare the simple small-signal model of the previous trick with one with a source resistance:

![Small-Signal Tricks Diagram](image)

In the left circuit, $v_{be} = v_{in}$, and so the output current is simply

$$i_{out} = g_m v_{be} = g_m v_{in}.$$  

However, in the right circuit, $v_{be}$ is divided such that $v_{be} = \frac{r_\pi}{R_S + r_\pi} v_{in}$. The output current will be similarly divided:

$$i_{out} = g_m v_{be} = \frac{r_\pi}{R_S + r_\pi} g_m v_{in}.$$  

Evidently, adding the source resistance decreased the amplification power of the transistor. To quantify this effect, we lump the division factor into $g_m$ to make an effective $g_m$:

$$g_{m,eff} = \frac{r_\pi}{R_S + r_\pi} g_m.$$  

Now, just as before, the amplification of the transistor is written as $i_{out} = g_{m,eff} v_{in}$. Thinking about effective $g_m$ can give quick intuition about the effect that dividing $r_\pi$ has on an amplifier’s gain.
**Boosting $R_E$** Consider the following circuit:

Let’s say we’re trying to find $r_{in}$, meaning we only care about the ratio between $v_{in}$ and $i_{in}$. To calculate $v_{in}$, we really only need to find $i_E$ since $v_{in} = i_{in}r_{\pi} + i_E R_E$. We find $i_E$:

$$i_E = i_{in} + g_m v_{be} = i_{in} + g_m i_{in}r_{\pi} = (\beta + 1)i_{in}.$$  

Interestingly, this is the same current that would flow through a resistor of size $(\beta + 1)R_E$ without the voltage-controlled current source. This allows us to transform the circuit as follows:

Here, $r_{in}$ is trivial to solve for. So, we see that a $g_m$ generator driving into $R_E$ effectively increases its resistance by a factor of $\beta$. Making this substitution can greatly simplify complicated small-signal models. As an example of this trick, consider a common emitter amplifier with emitter generation:
The small-signal model becomes

\[ r_{\text{in}} = (\beta + 1)R_E + r_{\pi}. \]

**Directly Wiggling \( g_m \)** Consider the following circuit:

Let’s say we’re trying to find \( r_{\text{out}} \). Because we’re driving \( v_{\text{be}} \), the voltage-controlled current source will output a current directly proportional to \( v_{\text{out}} \). And, since our output ports are directly across this current source, its current equals \( i_{\text{out}} \). Then, we simply have

\[ r_{\text{out}} = \frac{v_{\text{out}}}{i_{\text{out}}} = \frac{v_{\text{out}}}{g_m v_{\text{out}}} \Rightarrow r_{\text{out}} = \frac{1}{g_m}. \]
The intuition behind this is that we directly control the wiggle on \( v_{be} \), which \( g_m \) is related to. So, \( r_{out} \) depends directly on \( g_m \).

As an example, consider finding \( r_{out} \) of an emitter follower:

\[
\begin{align*}
V_{CC} & \quad v_{in} \\
& \quad v_{be} \\
V_B & \quad \downarrow \quad R_E \\
& \quad v_{out}
\end{align*}
\]

The small-signal model for finding \( r_{out} \) (ignoring \( r_o \)) is

\[
\begin{align*}
& \quad r_\pi \quad + \quad v_{be} \\
& \quad R_E \\
& \quad v_{out} \\
& \quad v_{be} \quad + \quad R_E \\
& \quad r_\pi \quad g_m v_{be} \\
& \quad v_{out} \\
& \quad v_{be} \quad + \quad r_\pi \quad g_m v_{be} \quad v_{out}
\end{align*}
\]

We can assume that very little current flows through \( r_\pi \) as compared with the voltage-controlled current source. We can also use the "flipping direction" trick so that the voltage-controlled current source points down and \( v_{be} \) is flipped. Then, this circuit perfectly matches the circuit for wiggling \( g_m \), and we find

\[
r_{out} \approx \frac{1}{g_m}.
\]

**Left-Right Pattern for \( r_{in} \)** Consider the following circuit:

\[
\begin{align*}
& \quad i_{in} \\
& \quad v_{in} \quad + \quad g_m v_{be} \quad R_r \\
& \quad v_{be} \quad \downarrow \quad + \quad \downarrow \quad v_{be} \quad \downarrow \quad R_i \\
& \quad + \quad - \quad - \quad -
\end{align*}
\]
In this topology, we call $R_l$ the left resistor and $R_r$ the right resistor. We solve for $r_{in}$ for this circuit:

$$v_{be} = i_{in} R_l$$

(1)

**KCL @ top node:**

$$i_{in} = -g_m v_{be} + \frac{v_{in} - v_{be}}{R_r}$$

Substituting in (1):

$$i_{in} = -g_m i_{in} R_l + \frac{v_{in} - i_{in} R_l}{R_r}$$

Multiply through by $R_r$:

$$R_r i_{in} = -g_m i_{in} R_l R_r + v_{in} - i_{in} R_l$$

$$\Rightarrow v_{in} = i_{in} (R_l + R_r + g_m R_l R_r)$$

(2)

Substituting in (2):

$$r_{in} = \frac{v_{in}}{i_{in}} \Rightarrow r_{in} = R_l + R_r + g_m R_l R_r$$

This pattern shows up surprisingly often, allowing the input resistance to be read off instead of derived. For example, consider a simple emitter follower. If we ignore any source and load resistance, we have:

The small signal model becomes

$r_\pi$ is in series with the bottom half of the circuit, so we can simply switch these two halves:
Using the left-right trick for $r_{in}$, we find by inspection that

$$r_{in} = r_{\pi} + R_E \parallel r_o + g_m r_{\pi} R_E \parallel r_o = (\beta + 1)(R_E \parallel r_o) + r_{\pi}.$$