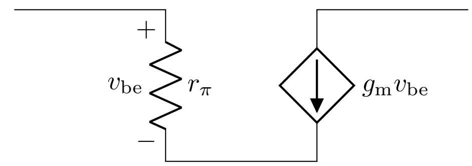
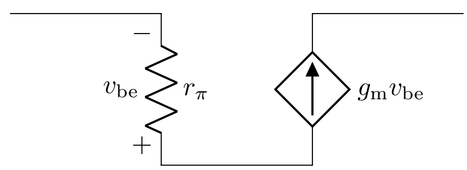
Small-Signal Tricks

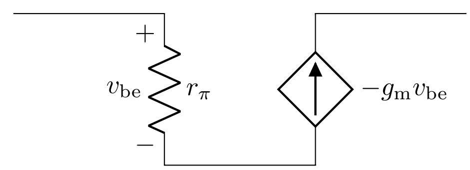
Flipping Direction Consider the following simple small-signal model:



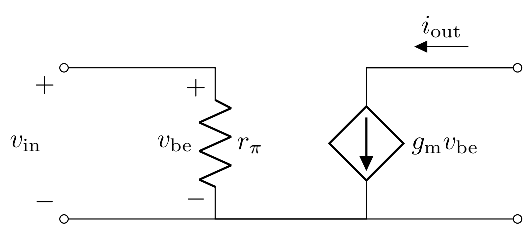
Sometimes, it's convenient to flip the direction of the voltage-controlled current source (for instance, to match one of the other tricks in this handout). In this case, v_{be} must be flipped too, either on r_π or in the voltage-controlled current source's equation:



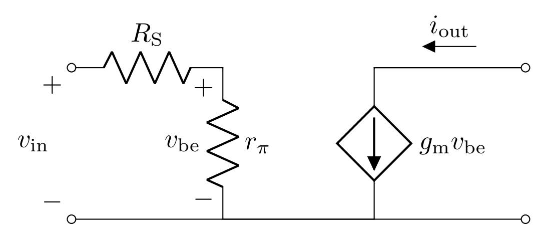
OR



Effective $\mathbf{g}_{\mathrm{m}}$ Compare the simple small-signal model of the previous trick with one with a source resistance:



VS.



In the left circuit, $v_{\mathrm{be}} = v_{\mathrm{in}}$, and so the output current is simply

$$i_{\mathrm{out}} = g_{\mathrm{m}}v_{\mathrm{be}} = g_{\mathrm{m}}v_{\mathrm{in}}.$$

However, in the right circuit, v_{be} is divided such that $v_{\mathrm{be}} = \frac{r_\pi}{R_{\mathrm{S}}+r_\pi}v_{\mathrm{in}}$. The output current will be similarly divided:

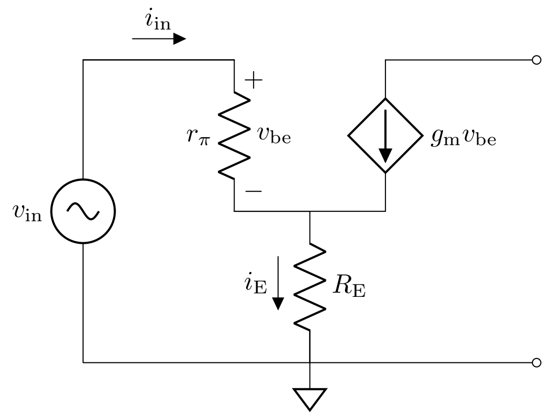
$$i_{\mathrm{out}} = g_{\mathrm{m}}v_{\mathrm{be}} = \frac{r_\pi}{R_{\mathrm{S}}+r_\pi}g_{\mathrm{m}}v_{\mathrm{in}}.$$

Evidently, adding the source resistance decreased the amplification power of the transistor. To quantify this effect, we lump the division factor into g_{m} to make an effective g_{m}:

$$g_{\mathrm{m,eff}} = \frac{r_\pi}{R_{\mathrm{S}}+r_\pi}g_{\mathrm{m}}.$$

Now, just as before, the amplification of the transistor is written as $i_{\mathrm{out}} = g_{\mathrm{m,eff}}v_{\mathrm{in}}$. Thinking about effective g_{m} can give quick intuition about the effect that dividing r_π has on an amplifier's gain.

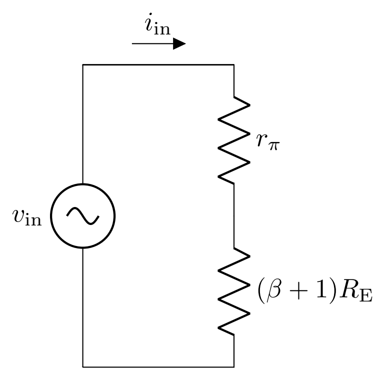
Boosting $\mathbf{R}_{\mathrm{E}}$ Consider the following circuit:



Let's say we're trying to find r_{in}, meaning we only care about the ratio between v_{in} and i_{in}. To calculate v_{in}, we really only need to find i_{E} since $v_{\text{in}} = i_{\text{in}} r_\pi + i_{\text{E}} r_{\text{E}}$. We find i_{E}:

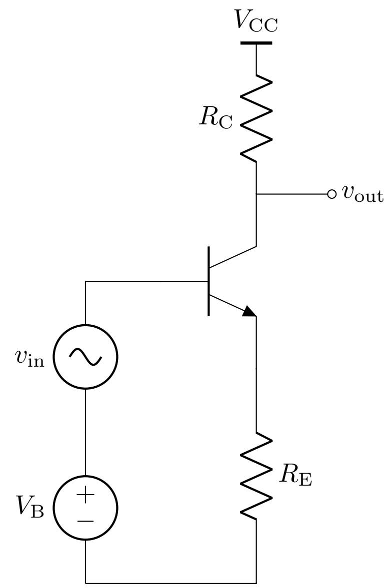
$$i_{\text{E}} = i_{\text{in}} + g_{\text{m}} v_{\text{be}} = i_{\text{in}} + g_{\text{m}} i_{\text{in}} r_\pi = (\beta + 1) i_{\text{in}}.$$

Interestingly, this is the same current that would flow through a resistor of size $(\beta + 1)R_{\text{E}}$ without the voltage-controlled current source. This allows us to transform the circuit as follows:

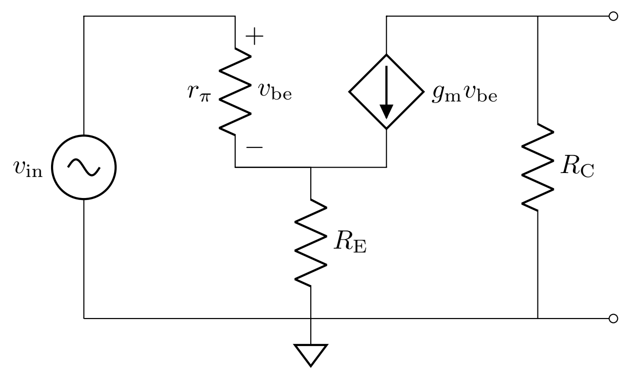


Here, r_{in} is trivial to solve for. So, we see that a g_m generator driving into R_{E} effectively increases its resistance by a factor of *beta*. Making this substitution can greatly simplify complicated small-signal models.

As an example of this trick, consider a common emitter amplifier with emitter generation:



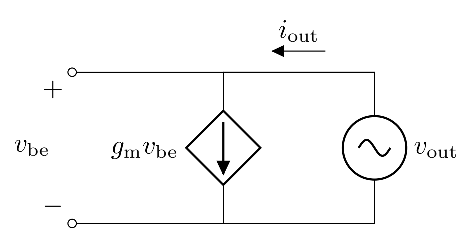
The small-signal model becomes



R_C doesn't affect r_{in}, so this circuit is the same as that at the beginning of this trick for finding r_{in}. Making the $(\beta + 1)R_E$ substitution, we find

$$r_{in} = (\beta + 1)R_E + r_\pi.$$

Directly Wiggling $\mathbf{g}_m$ Consider the following circuit:

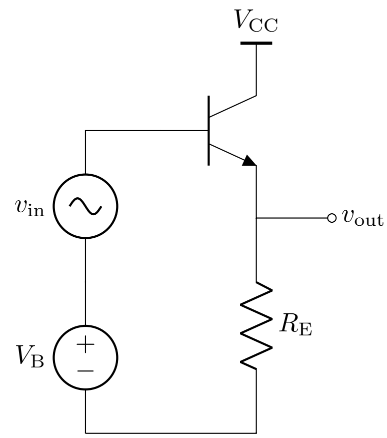


Let's say we're trying to find r_{out}. Because we're driving v_{be}, the voltage-controlled current source will output a current directly proportional to v_{out}. And, since our output ports are directly across this current source, its current equals i_{out}. Then, we simply have

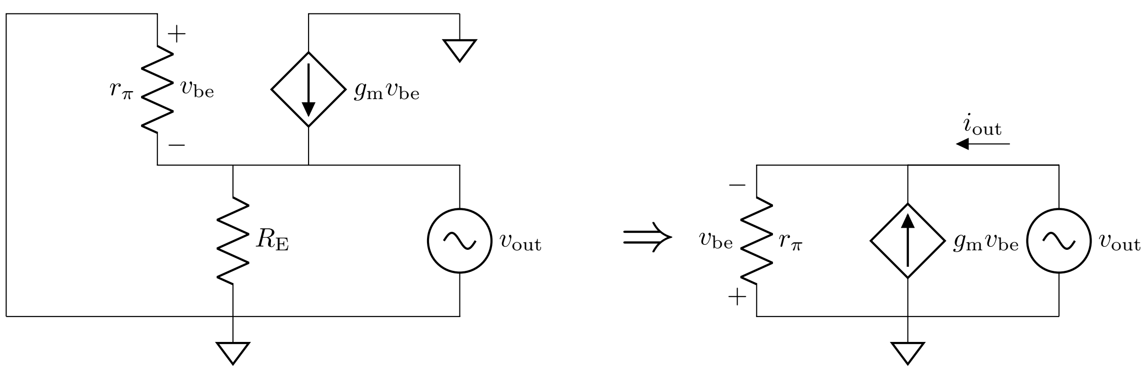
$$r_{out} = \frac{v_{out}}{i_{out}} = \frac{v_{out}}{g_m v_{out}} \;\Rightarrow\; \boxed{r_{out} = \frac{1}{g_m}}.$$

The intuition behind this is that we directly control the wiggle on v_{be}, which g_{m} is related to. So, r_{out} depends directly on g_{m}.

As an example, consider finding r_{out} of an emitter follower:



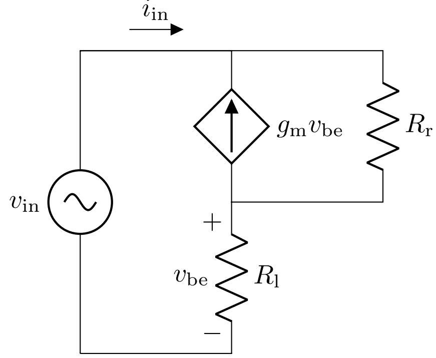
The small-signal model for finding r_{out} (ignoring r_{o})is



We can assume that very little current flows through r_π as compared with the voltage-controlled current source. We can also use the "flipping direction" trick so that the voltage-controlled current source points down and v_{be} is flipped. Then, this circuit perfectly matches the circuit for wiggling g_{m}, and we find

$$r_{\mathrm{out}} \approx \frac{1}{g_{\mathrm{m}}}.$$

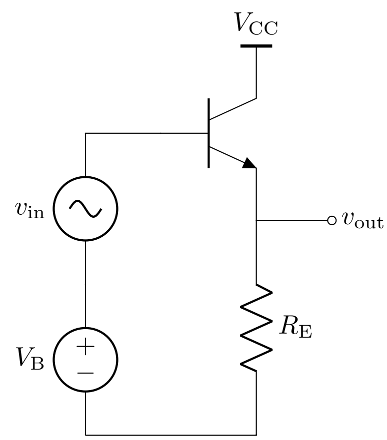
Left-Right Pattern for r$_{\mathrm{in}}$ Consider the following circuit:



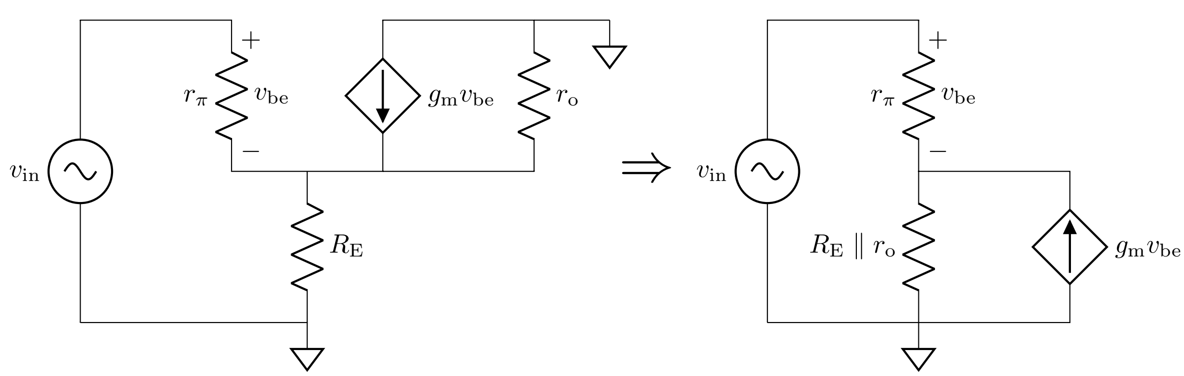
In this topology, we call R_l the left resistor and R_r the right resistor. We solve for r_in for this circuit:

$$
\begin{aligned}
& & v_\mathrm{be} &= i_\mathrm{in} R_\mathrm{l} & (1)\\
&\text{KCL @ top node:} & i_\mathrm{in} &= -g_\mathrm{m} v_\mathrm{be} + \frac{v_\mathrm{in} - v_\mathrm{be}}{R_\mathrm{r}} &\\
&\text{Substituting in (1):} & i_\mathrm{in} &= -g_\mathrm{m} i_\mathrm{in} R_\mathrm{l} + \frac{v_\mathrm{in} - i_\mathrm{in} R_\mathrm{l}}{R_\mathrm{r}} &\\
&\text{Multiply through by } R_\mathrm{r}: & R_\mathrm{r} i_\mathrm{in} &= -g_\mathrm{m} i_\mathrm{in} R_\mathrm{l} R_\mathrm{r} + v_\mathrm{in} - i_\mathrm{in} R_\mathrm{l} &\\
& & \Rightarrow v_\mathrm{in} &= i_\mathrm{in}(R_\mathrm{l} + R_\mathrm{r} + g_\mathrm{m} R_\mathrm{l} R_\mathrm{r}) & (2)\\
&\text{Substituting in (2):} & r_\mathrm{in} &= \frac{v_\mathrm{in}}{i_\mathrm{in}} \ \Rightarrow \ \boxed{r_\mathrm{in} = R_\mathrm{l} + R_\mathrm{r} + g_\mathrm{m} R_\mathrm{l} R_\mathrm{r}} &
\end{aligned}
$$

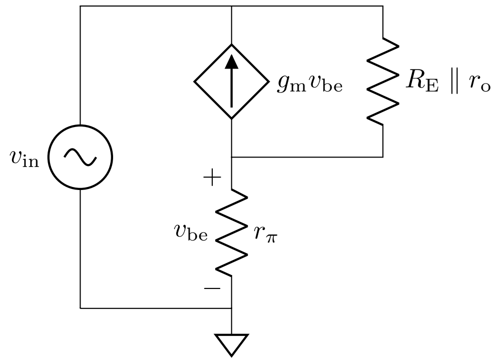
This pattern shows up surprisingly often, allowing the input resistance to be read off instead of derived. For example, consider a simple emitter follower. If we ignore any source and load resistance, we have:



The small signal model becomes



r_π is in series with the bottom half of the circuit, so we can simply switch these two halves:



Using the left-right trick for r_{in}, we find by inspection that

$$r_{\mathrm{in}} = r_\pi + R_{\mathrm{E}} \parallel r_{\mathrm{o}} + g_{\mathrm{m}} r_\pi R_{\mathrm{E}} \parallel r_{\mathrm{o}} = (\beta + 1)(R_{\mathrm{E}} \parallel r_{\mathrm{o}}) + r_\pi.$$