Diff amp review
- Degeneration
- Extends input range
- Equalizer
- diff gnd @ 2/2

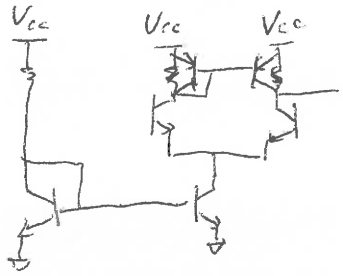
Sensitivity & Current mirrors
- Mirror current varies w/ supply
- Define sensitivity
- Widlar Reference

Supply Invariant Bias currents
- Principle: make a current from what you care about
- V_{be} reference
- self-biasing & startup

Bandgap References
- Why ~ regulators
- CTAT & PTAT & Cartoon
- Quick implementation
- $V_{out} = 1.26\ V$

We've been looking @ amplifiers biased & loaded by current mirrors

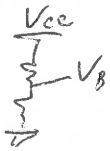
e.g.:



("diode reference")

- Current source biasing enables high gain & high swing active loads
- Enables high CMRR diff amp

Before we biased w/ voltage dividers



("divider reference")

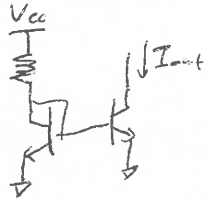
- ISSUE: ~~supply~~ bias point <u>very</u> sensitive to supply voltage
  ↳ voltage input range on op-amps often spans 10s of V.

- want to quantify how bad this is, do so w/ a measure called sensitivity

$$S^{\text{of } Y(X)}_{\text{to } X} \equiv S^{Y}_{X} \equiv \lim_{\Delta X \to 0} \frac{\Delta Y/Y}{\Delta X/X} = \frac{X}{Y}\frac{\partial Y}{\partial X}$$

← fractional change of Y w/r.t. X

eg: divider reference in simple mirror



$$S^{I_{out}}_{V_{cc}} = \frac{V_{cc}}{I_{out}}\frac{\partial}{\partial V_{cc}}\left[\left(\frac{\beta}{\beta+2}\right)\cdot\left(\frac{V_{cc}-V_{be,on}}{R}\right)\right]$$

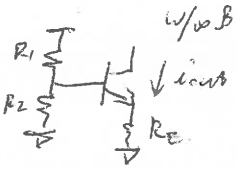
$$= \frac{\frac{\beta}{\beta+2}\cdot\frac{V_{cc}}{R}}{\left(\frac{\beta}{\beta+2}\right)\left(\frac{V_{cc}-V_{be,on}}{R}\right)}$$

$$= \frac{1}{\left(1-\frac{V_{be,on}}{V_{cc}}\right)} \approx 1$$

very sensitive!

<u>you try</u>

$S^{I_{out}}_{V_{cc}}$.

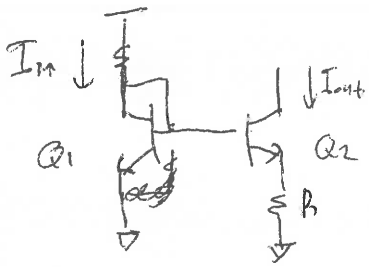


$$I_{out} = \left(\frac{R_2}{R_1+R_2}V_{cc} - V_{be,on}\right)\cdot\frac{1}{R_E}$$

$$\frac{\partial I_{out}}{\partial V_{cc}} = \frac{R_2}{R_E}\cdot\frac{1}{R_1+R_2}$$

$$S^{I_{out}}_{V_{cc}} = \frac{1}{\left(1-\left(1+\frac{R_1}{R_2}\right)\frac{V_{be,on}}{V_{cc}}\right)} \approx 1$$

Can make a mirror less sensitive to I_{in} using different V_{be}
- eg widlar ~~reference~~ current source



Interested in $I_{out}(I_{in})$

- $v_{be1} = v_{be2} + I_{out} R \cdot \frac{\beta}{\beta+1}$
- $I_{s1} e^{v_{be1}/\phi_T} = i_{C1} \rightarrow v_{be1} = \phi_T \ln \frac{i_{C1}}{I_{s1}}$
- $\phi_T \ln \frac{I_{in}}{I_{out}} \approx I_{out} R$

- Note can't assume 0.7V here
- Need an $1/\beta$ for V_{be} (bug last lecture)

- Transcendental, so let simulators solve exactly
- But relatively insensitive to I_{in} ~ logarithmically compressed
- called current source for that reason & not current mirror

Sensitivity ~ $\phi_T \frac{\partial}{\partial V_{cc}} \ln \frac{I_{in}}{I_{out}} = R \frac{\partial I_{out}}{\partial V_{cc}}$

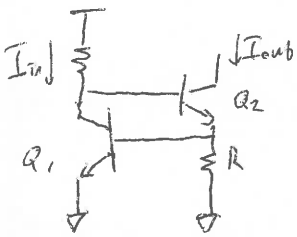
$$\phi_T \left(\frac{1}{I_{out}} \frac{\partial I_{in}}{\partial V_{cc}} - \frac{I_{in}}{I_{out}^2} \frac{\partial I_{out}}{\partial V_{cc}} \right) = R \frac{\partial I_{out}}{\partial V_{cc}}$$

$$\frac{\partial I_{out}}{\partial V_{cc}} = \left(\frac{1}{1 + \frac{I_{out} R}{\phi_T}} \right) \cdot \frac{I_{out}}{I_{in}} \cdot \frac{\partial I_{in}}{\partial V_{cc}}$$

$$S_{V_{cc}}^{I_{out}} = \frac{V_{cc}}{I_{out}} \cdot \frac{\partial I_{out}}{\partial V_{cc}} = \left(\frac{1}{1 + I_{out} R/\phi_T} \right) \cdot \underbrace{S_{V_{cc}}^{I_{in}}}_{\approx 1} \approx \left(\frac{1}{1 + \frac{I_{out} R}{\phi_T}} \right)$$

w/ 1mA I_{out} & R = 1k (high V_{cc}!)
1/40 better!

You try V_{be} reference



$I_{out} = v_{be1}/R$

$= \frac{\phi_T}{R} \ln \frac{I_{in}}{I_{s1}}$

$\frac{\partial I_{out}}{\partial V_{cc}} = \frac{\phi_T}{R} \cdot \frac{I_{s1}}{I_{in}} \cdot \cancel{\frac{V_{cc} - V_{be,on}}{R}} \cdot \frac{1}{I_{s1}} \cdot \frac{\partial I_{in}}{\partial V_{cc}}$

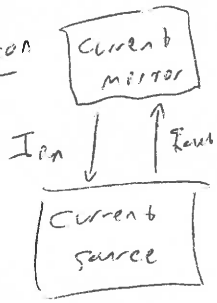
- Note: pretty good vs V_{cc}
$S = \frac{26\,mV}{700\,mV} = 3.7\%$

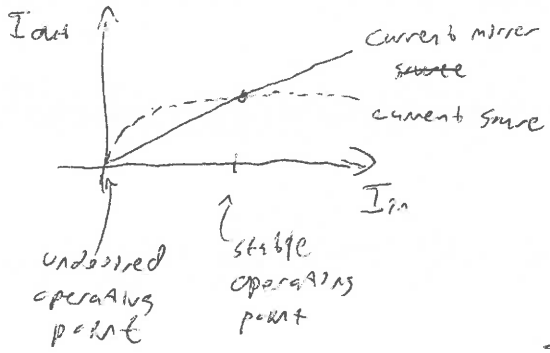
$S_{V_{cc}}^{I_{out}} = \frac{\phi_T}{R} \cdot \frac{1}{I_{in}} \cdot \frac{\partial I_{in}}{\partial V_{cc}} \cdot \frac{V_{cc}}{I_{out}} = \frac{\phi_T}{v_{be,on}} \cdot S_{V_{cc}}^{I_{in}}$

b/c $I_{out} R = v_{be,on}$
& $\frac{V_{cc}}{I_{in}} \frac{\partial I_{in}}{\partial V_{cc}} = S_{V_{cc}}^{I_{in}}$

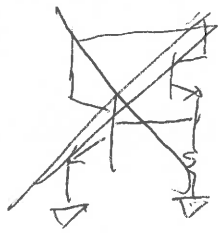
Further improve supply insensitivity w/ self-biasing (no dependence on V_{cc} - R at all)

carbon

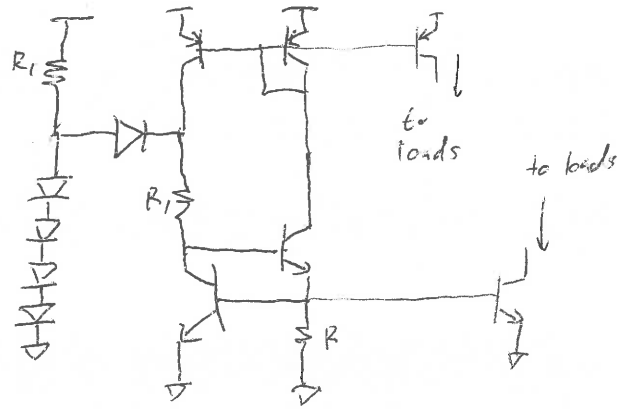




- Need a startup circuit to kick to correct op. pt.



- 4 diode drops vs. 3 guarantees current injection @ startup until $V_{R1} = V_{be,on}$
- big Rx rev. biases diode if $R_2 \cdot I_{in} > 2V_{be,on}$



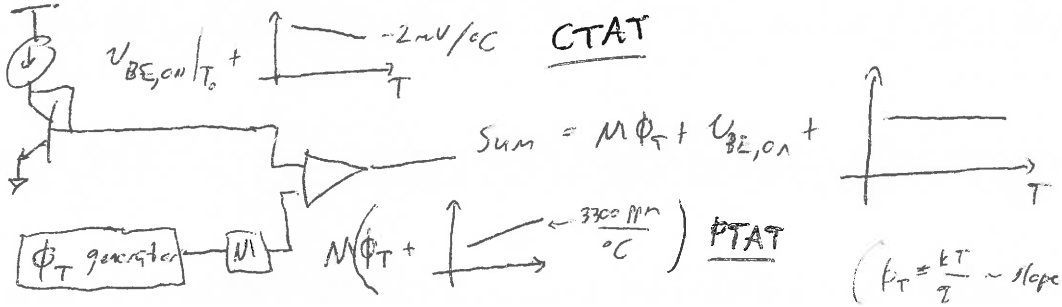
Look for corners

Temperature Insensitivity

- measured w/ Temperature coefficient $TC^{I_{out}} = \frac{1}{I_{out}} \frac{\partial I_{out}}{\partial T}$
  - fractional change in output per degree C (not sensitivity b/c don't care about fractional temp)
  - shows up a lot in resistors (~~[illegible]~~ 1000 ppm/°C) $\left(\frac{1}{R}\frac{\partial R}{\partial T}\right)$

→ Various tricks to adjust temperature in self-biased circuits

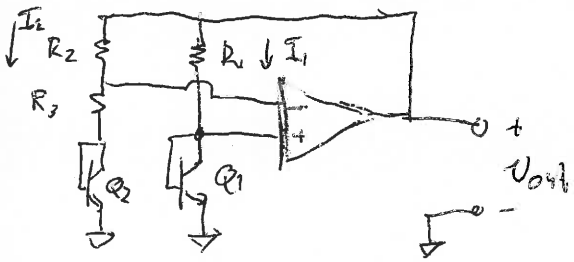
- $V_{be} \propto T$ (-2mV/°C) & ~~[illegible]~~ $\propto T$, so tend to cancel in wider current references. Bad in V_{BE}
- But sometimes want supply & temp invariant voltages (eg: regulator reference)
- so we'll look @ bandgap reference ~ an iconic reference



- Cancel out temp dependence!

($\phi_T = \frac{kT}{q}$ ~ slope = $\frac{k}{q}$; $V_{be,on} = \phi_T \ln I_1/I_s$ related to I_s)

Band gap reference does this in a very literal way



– op-amp forces $V_{R1} = V_{R2}$

– so $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

– ϕ_T generating circ can get it from ΔV_{be}

- $V_{R3} = \underbrace{V_{be1} - V_{be2}}_{\Delta V_{be}} = \phi_T \ln \frac{I_1}{I_{S1}} - \phi_T \ln \frac{I_2}{I_{S2}} = \phi_T \ln \frac{R_2}{R_1} \frac{I_{S2}}{I_{S1}}$

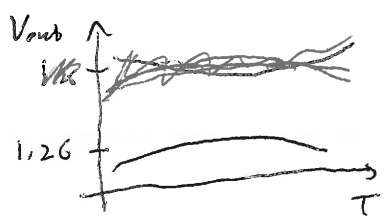
- same current in R_2 & R_3, so $V_{R2} = \frac{R_2}{R_3} V_{R3}$

Finally, do KVL, $V_{out} = V_{be2} + V_{R2} + V_{R3}$

$$= V_{be2} + \left(1 + \frac{R_2}{R_3}\right) \phi_T \ln \frac{R_2}{R_1} \frac{I_{S2}}{I_{S1}}$$

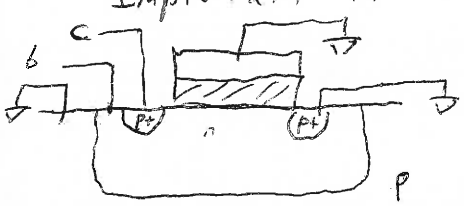
$$= V_{be2} + M\phi_T \quad \text{where} \quad M = \left(1 + \frac{R_2}{R_3}\right) \ln \frac{R_2}{R_1} \frac{I_{S2}}{I_{S1}}$$

– Generally have an output $V_{BG} = 1.26V$ ~ tenuous relationship w/ band gap of silicon in eV



– parabolic
– slightly concave down
– varies a few mV over 100°C

– Implement in CMOS using parasitic substrate BJT



pnpn device