

Exact EF

- R_{in} & A_v easy analysis
- Z_{out} & negative values

OCTC

- cascade w/ bias currs
- pole derive
- picking caps
- Test on CE (they)

SCTC

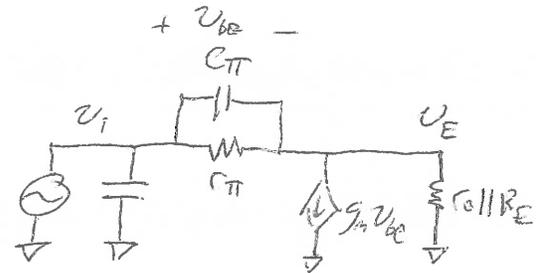
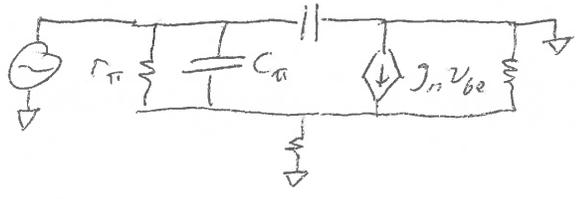
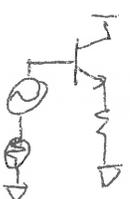
- If we get there
- $\omega = \sum \frac{1}{C_i R_{i, short cap}}$

Lab Manual Expectations

- radl to row vs. ARRAY cap
- re evaluate model to make it plausibly match
- Bonus points ①

- We're doing better approx techniques today
 - Need to do one more exact analysis first for EF

↳ EF is kind of approximation resistant b/c it has a zero
 ↳ C_{π} is grounded which makes life easier



$$Z_{\pi} = r_{\pi} \parallel \frac{1}{C_{\pi} s} = \frac{r_{\pi}}{1 + r_{\pi} C_{\pi} s}$$

Follow same analysis as DC, but use Z_{π} in place of r_{π}

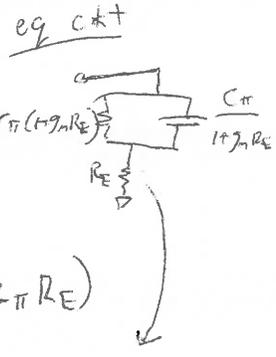
~~$v_E = (i_{\pi} + g_m v_{be}) R_E$~~

$$v_E = (i_{\pi} + g_m i_{\pi} Z_{\pi}) R_E$$

$$v_i = v_{be} + v_E = i_{\pi} (R_E + Z_{\pi} + g_m Z_{\pi} R_E)$$

$$Z_{in} = \frac{1}{C_{\pi} s} \parallel \frac{v_i}{i_{\pi}} = \frac{1}{C_{\pi} s} \parallel R_E + \frac{r_{\pi}}{1 + r_{\pi} C_{\pi} s} + \frac{\beta R_E}{1 + r_{\pi} C_{\pi} s}$$

$$A_v = \frac{v_E}{v_i} = \frac{R_E + g_m Z_{\pi} R_E}{R_E + Z_{\pi} + g_m Z_{\pi} R_E} = \frac{R_E + \beta R_E + r_{\pi} C_{\pi} s R_E}{R_E + \beta R_E + r_{\pi} + r_{\pi} C_{\pi} s R_E} = \frac{\beta + 1}{\beta + 1 + \frac{r_{\pi}}{R_E}} \cdot \frac{1 + \frac{r_{\pi} C_{\pi} s}{\beta + 1}}{1 + \frac{r_{\pi} C_{\pi} s}{\beta + 1 + \frac{r_{\pi}}{R_E}}}$$



Commentary

- Has a zero
- pole & zero very high
- $f_z \approx \frac{g_m}{C_{\pi}} > f_T$

- Bode - C_{π} limits BW w/ R_E

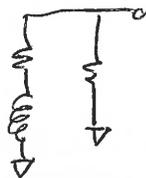
For Z_{out}
$$v_E = \frac{v_E}{r_o} + \frac{v_E}{Z_{\pi} + R_S} + g_m v_E \frac{Z_{\pi}}{Z_{\pi} + R_S}$$

$$Z_{out} = \frac{v_E}{i_E} = r_o \parallel \frac{Z_{\pi} + R_S}{1 + g_m Z_{\pi}} \approx \frac{r_o + R_S r_o C_{\pi} s + R_S}{r_o C_{\pi} s + 1 + \beta} = \frac{R_S + r_o}{\beta + 1} \cdot \frac{1 + \frac{R_S r_o C_{\pi} s}{R_S + r_o}}{1 + \frac{r_o C_{\pi} s}{\beta + 1}}$$

- In reality C_{π} in parallel w/ R_S , ignored here, more poles
- Zero & pole, like gain, but locations can swap depending on R_S
- Big $R_S \rightarrow$ zero ^{much} before pole \rightarrow inductive Z_{out} (won't unstable!)

(Large capacitive loads?)

eq.ckt



OCTC

- Miller was not conservative, also specific to CE
- Open circuit the constants from MIT in 60's, better
- Approx assumes form of $X_{for} f_{\Delta}$

Real CE: $H(s) = A_v \frac{(1 - s/z)}{(1 - s/p_1)(1 - s/p_2)}$; Miller: $H(s) = A_v \frac{1}{1 - s/p_M}$ lose info, use ZTH of C_{π} to get p_M

$$OCTC: H(s) = A_v \frac{1}{(1 - s/p_1)(1 - s/p_2) \dots (1 - s/p_n)} = A_v \frac{1}{1 + \underbrace{\left(\frac{1}{p_1} + \frac{1}{p_2} + \dots + \frac{1}{p_n}\right)}_{\text{same as } \tau_1 + \tau_2 + \dots} s + \dots + \underbrace{\left(\frac{1}{p_1 p_2 \dots p_n}\right)}_{\text{same as } \tau_1 \tau_2 \dots \tau_n} s^n}$$

- OCTC Assumes 1 term is dominant, ~~dominant~~ b/c sb s close to BW τ_s small
- Means higher $s^2 \rightarrow s^n$ terms irrelevant vs s term

so have
$$H(s) \approx A_v \frac{1}{1 + (\tau_1 + \tau_2 + \dots + \tau_n) s}$$

OK, this approx is great, but we don't know τ_1, τ_2 , etc.

↳ Remarkably can find these w/ a ckt track

↳ sum of $\sum \tau_i = \sum OCTC_i$ where open ckt time const is:

↳ $OCTC_i = C_i (R_{TH} \text{ seen from } C_i)$ for each cap in ckt.
w/ other caps open

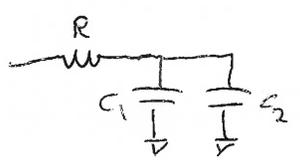
- want caps
- when fails or why

"pole consists"

↳ Tricky, more labor

↳ OCTC not associated w/ 1 pole

Example (trivial)



Exact

$$\tau = R(C_1 + C_2)$$

OCTC

$$OCTC_1 = C_1 R$$

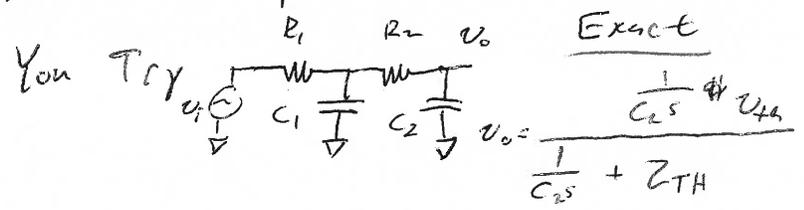
$$OCTC_2 = C_2 R$$

$$\sum OCTC = R(C_1 + C_2)$$

Perfect!

↳ Expect right for 1st order...

Less trivial example



Exact

$$u_0 = \frac{\frac{1}{C_2 s} \# u_{TH}}{\frac{1}{C_2 s} + Z_{TH}}$$

$$Z_{TH} = R_2 + R_1 \parallel \frac{1}{C_1 s}$$

$$= R_2 + \frac{R_1}{1 + R_1 C_1 s}$$

$$\frac{u_0}{u_1} = \frac{\frac{1}{C_2 s} - \frac{1}{1 + R_1 C_1 s}}{\frac{1}{C_2 s} + R_2 + \frac{R_1}{1 + R_1 C_1 s}}$$

$$N_{TH} = \frac{\frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} u_1 = \frac{u_1}{1 + R_1 C_1 s}$$

$$= \frac{1}{(1 + R_1 C_1 s) + R_2 C_2 s (1 + R_1 C_1 s) + R_1 C_2 s} = \frac{1}{1 + (R_1 + R_2) C_2 s + R_1 R_2 C_1 C_2 s^2 + \dots + R_1 C_1 s}$$

↳ $(1 + s/P_1)(1 + s/P_2)$

vs. OCTC

$$OCTC_1 = C_1 R_1$$

$$OCTC_2 = C_2 (R_1 + R_2)$$

$$3_{dB} @ (R_1 C_1 + (R_1 + R_2) C_2)^{-1}$$

- Got 1st order term!

- works great if $\tau_1 > \tau_2$

- systems often designed that way

+ Miller effect makes it likely

- what happens if poles are close?

$$\frac{1}{1 + (z_1 + z_2)s + z_1 z_2 s^2} \xrightarrow{\text{FRF}} \frac{1}{1 + \underbrace{(z_1 + z_2)\omega}_{-3dB, \text{ est}} + \underbrace{z_1 z_2 \omega^2}_{-dB, \text{ est}}}$$

= 1 by defn of OCTC

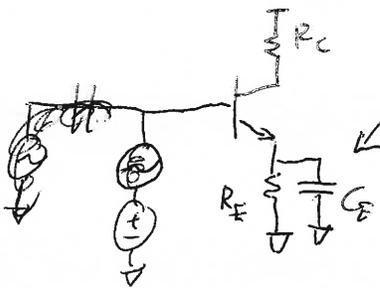
each of z_1 & z_2 must be < 1 , so product much smaller
 worst case, $z_1 = z_2 \rightarrow$ this is $\frac{1}{4}(z_1 + z_2)^2$

- Also, even if this doesn't give us the exact

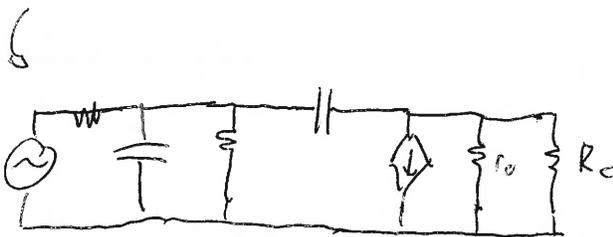
ω_{-3dB} , it does give us \rightarrow ① a good guess

\rightarrow ② design insight about which caps cause trouble

Let's apply this to a CE



- This cap causes a zero in addition to a pole
 - we assume an all-pole system w/ OCTC
 - short @ high freq, & causes problems, so we ignore
 - same w/ coupling caps (GETC next time)

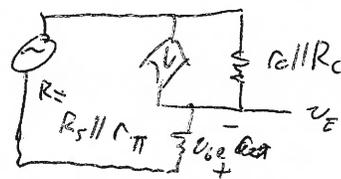


You guys find OCTC $_{\pi}$

$$= (R_s || r_{\pi}) C_{\pi}$$

- clearly the trouble cap
 - written in specific way
 $R_{sig} + R_{out} + g_m \cdot R_{sig} R_{out}$
 are generic

OCTC $_{\pi}$



$$i_E = g_m v_{be} + (v_{be} - v_E) / r_o || R_c$$

$$= -g_m r_{\pi} i_E + (v_{be} - r_{\pi} i_E) / r_o || R_c$$

$$i_E (1 + g_m r_{\pi} + r_{\pi}) r_o || R_c = v_{be}$$

$$R_{in} = r_{\pi} || R_s + r_{\pi} R_s + g_m r_{\pi} R_s r_o || R_c$$

Compare

Exact

$$z_1 = C_{\pi} R + C_{\mu} (R + R_c + g_m R R_c)$$

Miller

$$z_1 = C_{\pi} R + C_{\mu} (R + g_m R R_c)$$

We beat Miller!