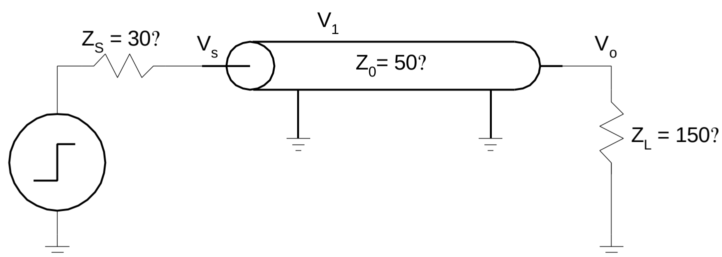
Introduction to Computer Engineering (E85)

Harris

Transmission Line Example

Given the transmission line shown below, calculate the voltages V_S, V_1, and V_o given a unit step on the input. Assume the wire is of length l = cT, where c is the speed of light so T is the flight time along the line. Let V_1 be measured one third of the way along the line. Predict the final voltage on V_S, V_1, and V_o.



Key Equations:

Reflection Coefficients (given in class)

Source: $?_S \;?\; \frac{Z_S \;?\; Z_0}{Z_S \;?\; Z_0}$ (EQ1)

Load: $?_L \;?\; \frac{Z_L \;?\; Z_0}{Z_L \;?\; Z_0}$ (EQ2)

Initial voltage on the line: $V_s \;?\; \frac{Z_0}{Z_S \;?\; Z_0}$ (EQ3)

Derivation: at time 0, the line looks like an impedance of Z0. So we have a voltage divider between the line and the source.

Voltage on line after wave arrives = Previous voltage + incident wave + reflected wave (EQ4)

Final Voltage: $V_S \;?\; V_1 \;?\; V_0 \;?\; \frac{Z_L}{Z_S \;?\; Z_L}$ (EQ5)

Derivation: after inifinite time has passed, all reflections have damped out. The final voltage at all points along the line (beginning, middle, and end) must be the same because the line looks like a perfect wire. So we just have a voltage divider between the load and source.

Analysis

At time 0, the voltage source steps from 0 to 1. We expect a wave to be driven into the transmission line and to reflect multiple times. After many flight times T have passed, we expect the reflections to have decayed to be negligible. At this time, the transmission line should look just like a perfect wire with no resistance. Therefore, the final voltage at all points along the line will be 0.83333, by (EQ5).

The reflection coefficients are –1/4 at the source and ½ at the load by (EQ1) and (EQ2).

Until this time has passed, however, we will see oscillations along the line. At time 0, a wave is incident into the transmission line, which was initially at 0 volts. We don't have time to know yet what is at the end of the transmission line, so the line maintains a ratio of voltage to current of Z0 and hence initially looks like a resistance of Z0 to ground. By the voltage divider equation of (EQ3), we get an initial voltage of 0.625 traveling down the line. From now on, we will keep track of what voltages are already on the line and what waves are traveling along the line. The line voltages are shown in the diagram on the next pages. Arrows between signals indicate the amplitude of the waves traveling along the line at any given time.

At T/3, the incident wave reaches V1, which becomes the initial value (0) plus the incident wave (0.625), or 0.625. Because the transmission line is uniform, there are no internal reflections. If the transmission line were nonuniform and had impedance changing with distance, there could be internal reflections which would complicate the behavior even more!

At T, the incident wave reaches the end of the line. The reflection coefficient (EQ2) is ½, so we get a reflected wave of (0.625*1/2)= 0.3125 caused by the impedance mismatch between the line and the load. The new voltage at V0 becomes the initial voltage (0) + incident wave (0.625) + reflected wave (0.3125) = 0.9375 by (EQ4).

At 5T/3, the wave reaches V1 again. The voltage becomes the initial value (0.625) plus the wave (0.3125) = 0.9375.

At 2T, the wave returns to Vs. The reflection coefficient is –1/4 (EQ1), so we get a reflected wave of (0.3125*(-1/4))= -0.078125. The voltage becomes the initial value (0.625) + the incident wave (0.3125) + the reflected wave (-0.078125), or 0.859375 by (EQ4).

At 7T/3, the wave reaches V1 again. The voltage becomes the initial value (0.9375) plus the wave (-0.078125) = 0.859375.

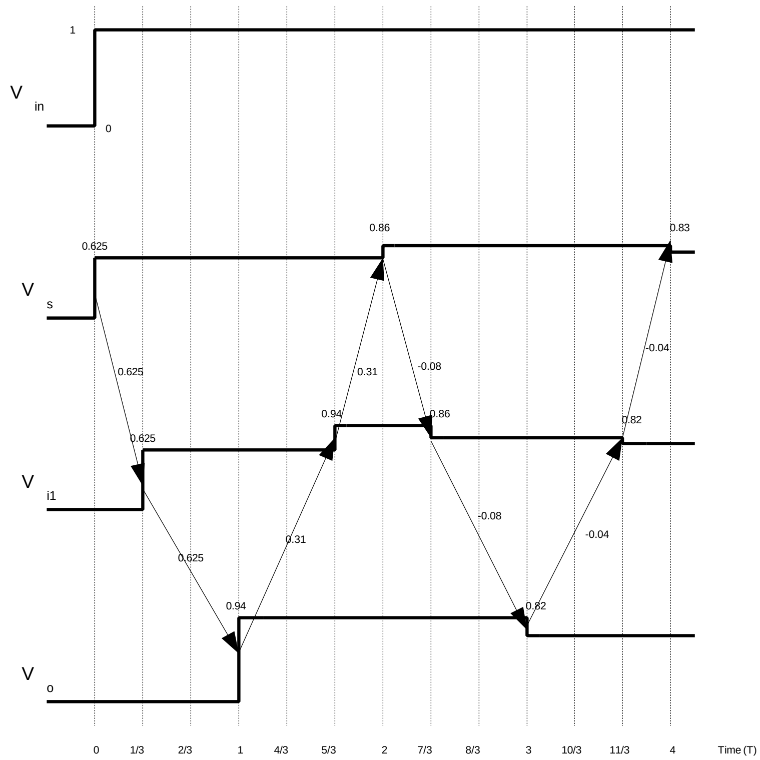
At 3T, the wave reaches the end of the line again (V0). We get a reflected wave of (-0.078125*(1/2))= -0.039063. Hence, the voltage at V0 becomes the initial value (0.9375)+ incident (-0.078125) + reflected (-0.039063) = 0.820313.

At 11T/3, the wave reaches V1 again. The voltage becomes the initial value (0.859375) + the wave (-0.039063) = 0.820313.

At 4T, the wave returns to Vs. We get a reflected wave of (-0.039063 * (-1/4)) = 0.009766. The voltage becomes the initial value (0.859375) + the incident (-0.039063) + the reflected (0.009766) = 0.830079. This is within 1% of the final value (0.83333) that we expect on the line. Waves will continue to reflect back and forth along the line, asymptotically approaching the final value.

Transmission line analysis is important for several reasons. If the load impedance is much higher than the line impedance, the voltage at the end of the line may initally spike well above the input voltage, potentially damaging the circuits at the end of the line. The oscillations along the line may cause voltage levels to become valid, then invalid, then valid again. And other circuits connected to stubs along the line may see strange oscillations. In fact, impedance discontinuities caused by theses stubs may introduce reflections of their own!

Waveforms



1
V in
0
0.86
0.83
0.625
V s
0.625
0.31
-0.08
-0.04
0.94
0.86
0.82
0.625
V i1
-0.08
0.31
-0.04
0.625
0.94
0.82
V o
0
1/3
2/3
1
4/3
5/3
2
7/3
8/3
3
10/3
11/3
4
Time (T)