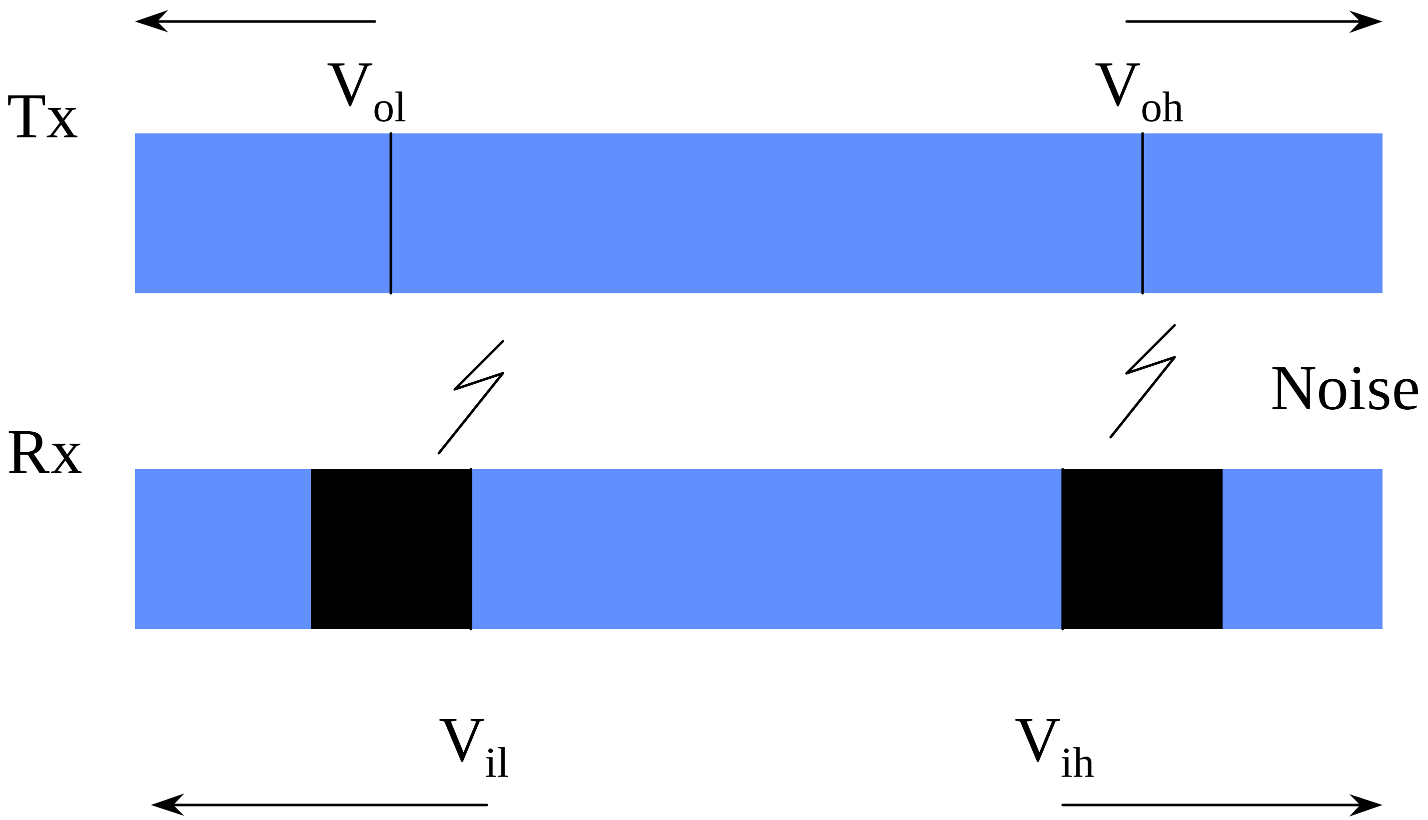
6.004 L13:
Introduction to the Physics of Communication

The Digital Abstraction Part 1: The Static Discipline



What is Information?

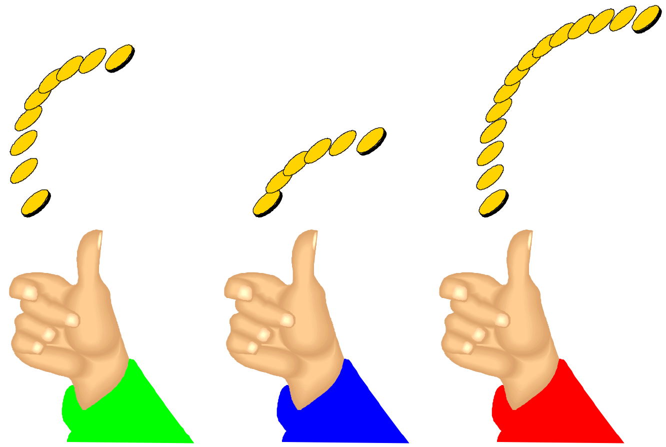
Information Resolves ____**Uncertainty**____

How do we measure information?

Error-Free data resolving 1 of 2 equally likely possibilities =

1 bit of information.

How much information now?



3 independent coins yield ___**3 bits**___ of information

# of possibilities = ___**8**___

How about N coins ?

N independent coins yield

bits = ______ N ______

of possibilities = ____ 2^N

What about Crooked Coins?

$P_{tail} = .25$

$P_{head} = .75$

$\# \text{ Bits} = - \sum p_i \log_2 p_i$

(about .81 bits for this example)

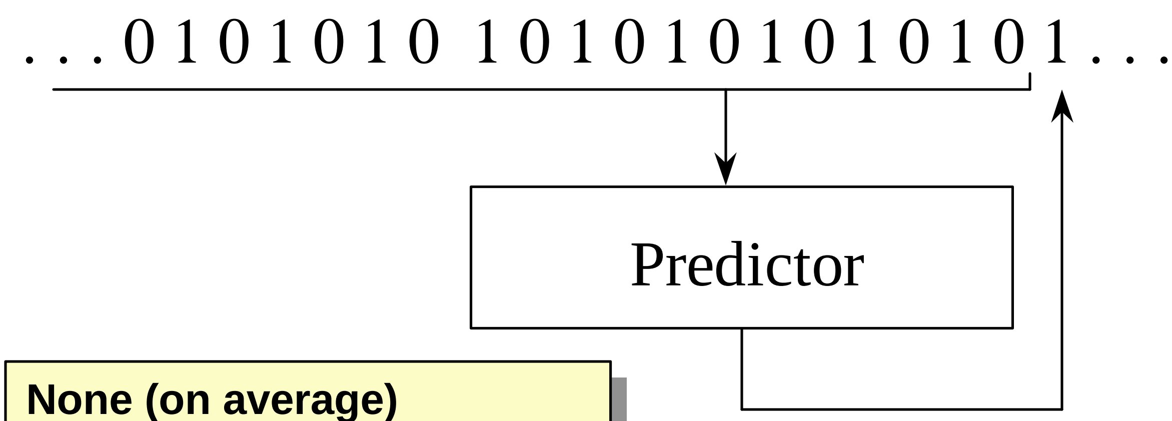
How Much Information ?

. . . 00000000000000000000000000000 . . .

None (on average)

How Much Information Now ?

. . . 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 . . .

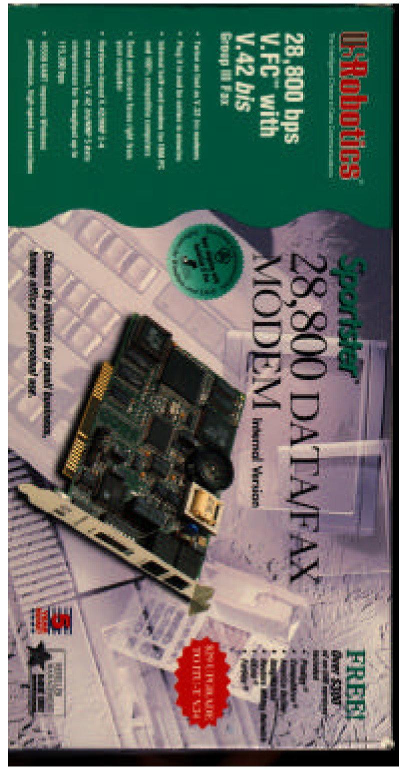


None (on average)

How About English?

- 6.JQ4 ij a vondurfhl co8rse wibh sjart sthdenjs.
- If every English letter had maximum uncertainty, average information / letter would be __ $\log_2(26)$ __
- Actually, English has only __ 2 __ bits of information per letter if last 8 characters are used as a predictor.
- English actually has __ 1 __ bit / character if even more info is used for prediction.

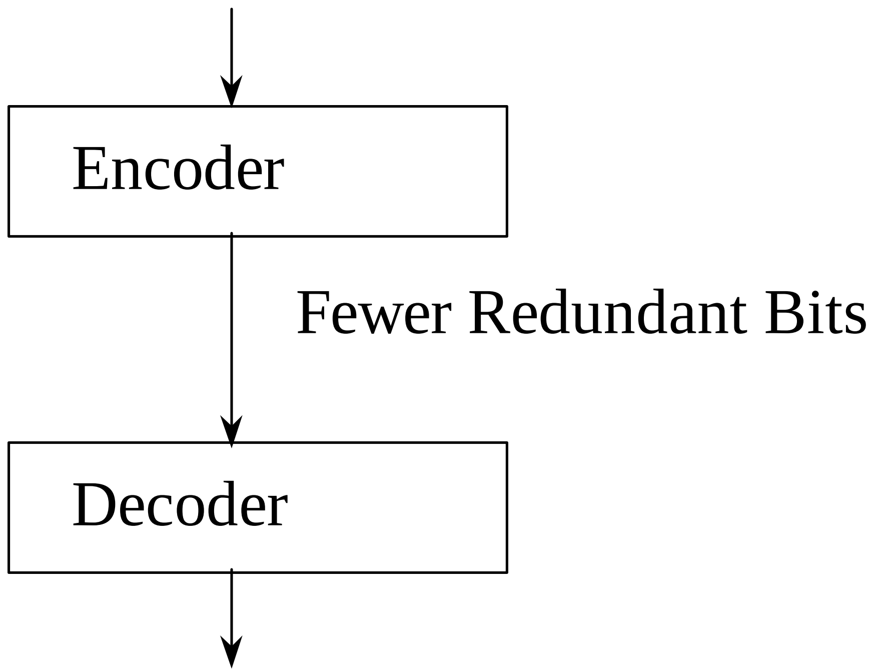
Why Do These Work?



Answer: They Lower ________ **Redundancy** ____

Data Compression

Lot's O' Redundant Bits

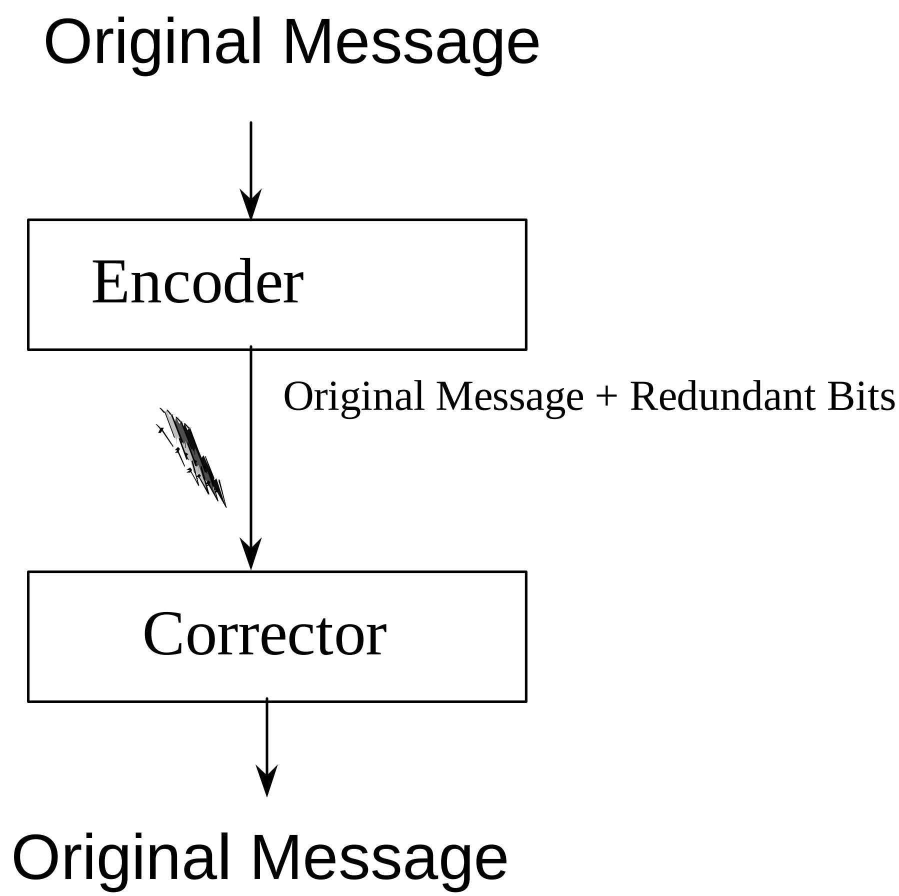


Lot's O' Redundant Bits

An Interesting Consequence:

- A Data Stream containing the most possible information possible (i.e. the least redundancy) has the statistics of ______ **Random Noise** ______ !!!!!

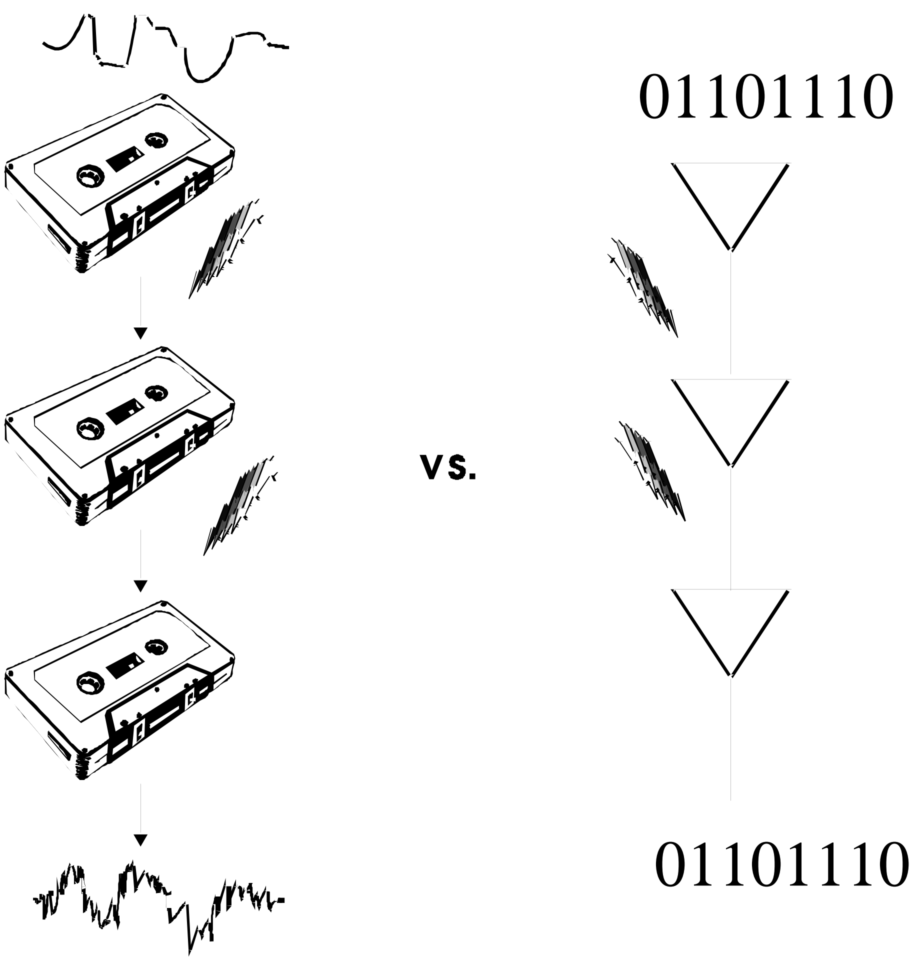
Digital Error Correction



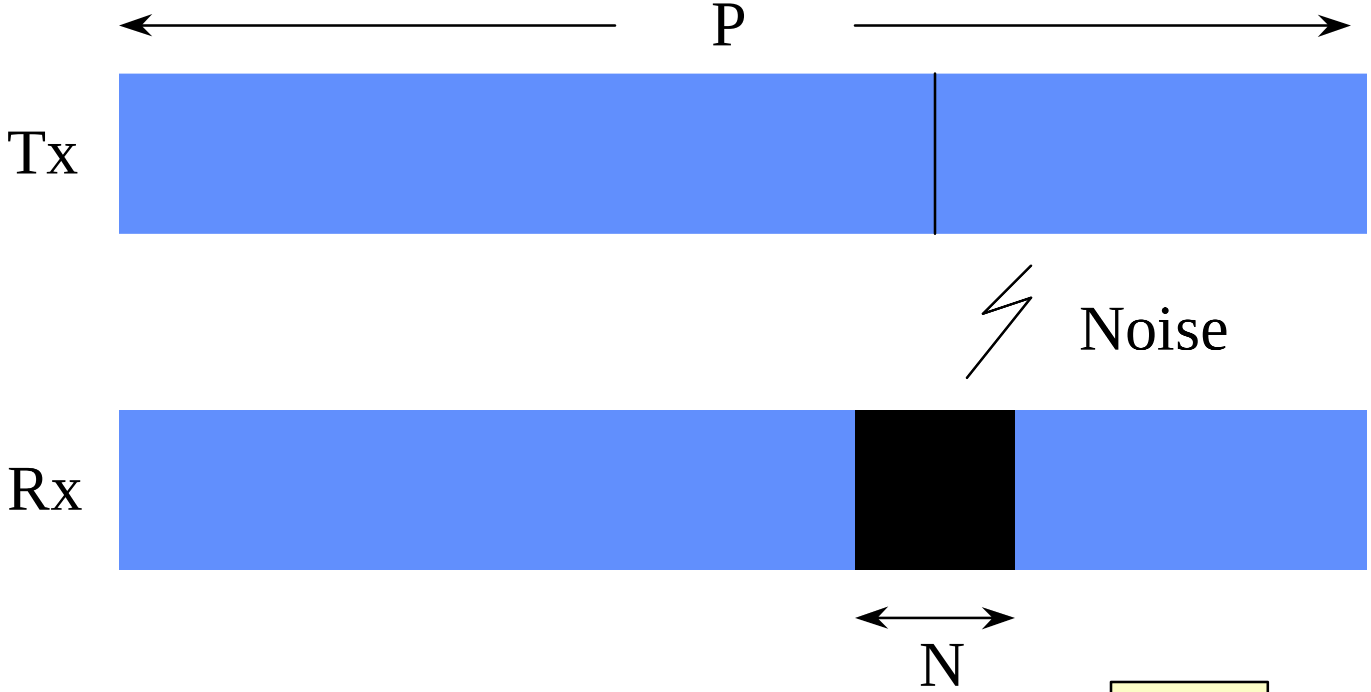
How do we encode digital information in an analog world?

Once upon a time, there were these aliens interested in bringing back to their planet the entire library of congress ...

The Effect of “ Analog” Noise



Max. Channel Capacity for Uniform, Bounded Amplitude Noise



Max # Error-Free Symbols = P/N

Max # Bits / Symbol = $\log_2(P/N)$

Max. Channel Capacity for Uniform, Bounded Amplitude Noise (cont)

P = Range of Transmitter's Signal Space
N = Peak-Peak Width of Noise
W = Bandwidth in # Symbols / Sec
C = Channel Capacity = Max. # of Error-Free Bits/Sec

C = $W \log_2(P/N)$

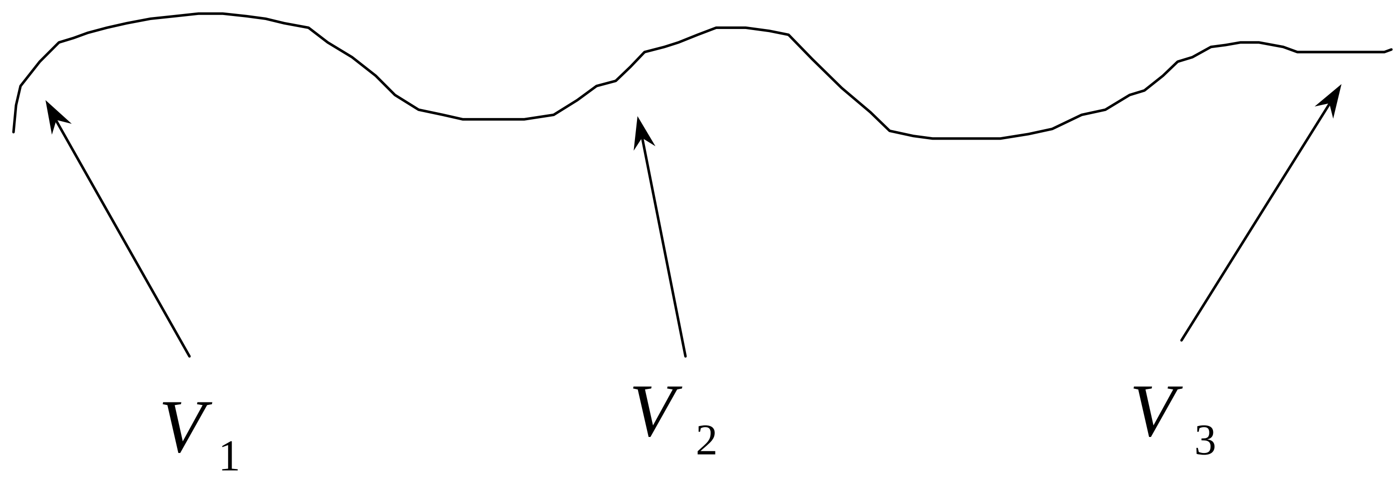
Note: This formula is slightly different for Gaussian noise.

Further Reading on Information Theory

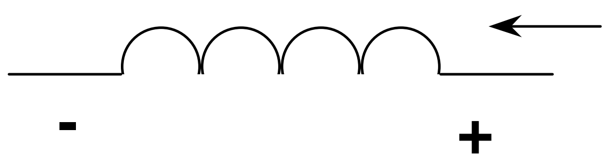
The Mathematical Theory of Communication, Claude E. Shannon and Warren Weaver, 1972, 1949.

Coding and Information Theory, Richard Hamming, Second Edition, 1986, 1980.

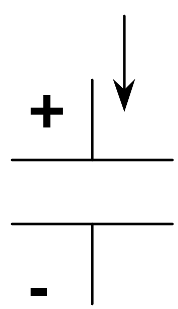
The mythical equipotential wire



But every wire has parasitics:

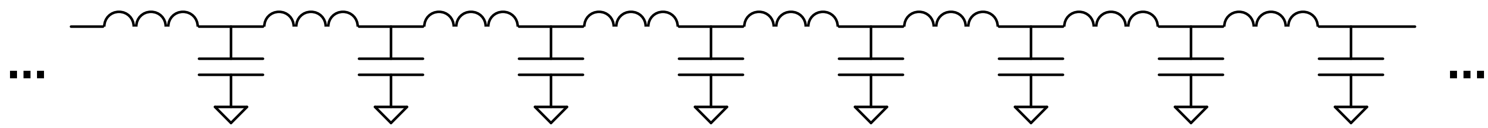


$$V = L\frac{dI}{dt}$$



$$I = C\frac{dV}{dt}$$

Why do wires act like transmission lines?



Signals take time to propagate

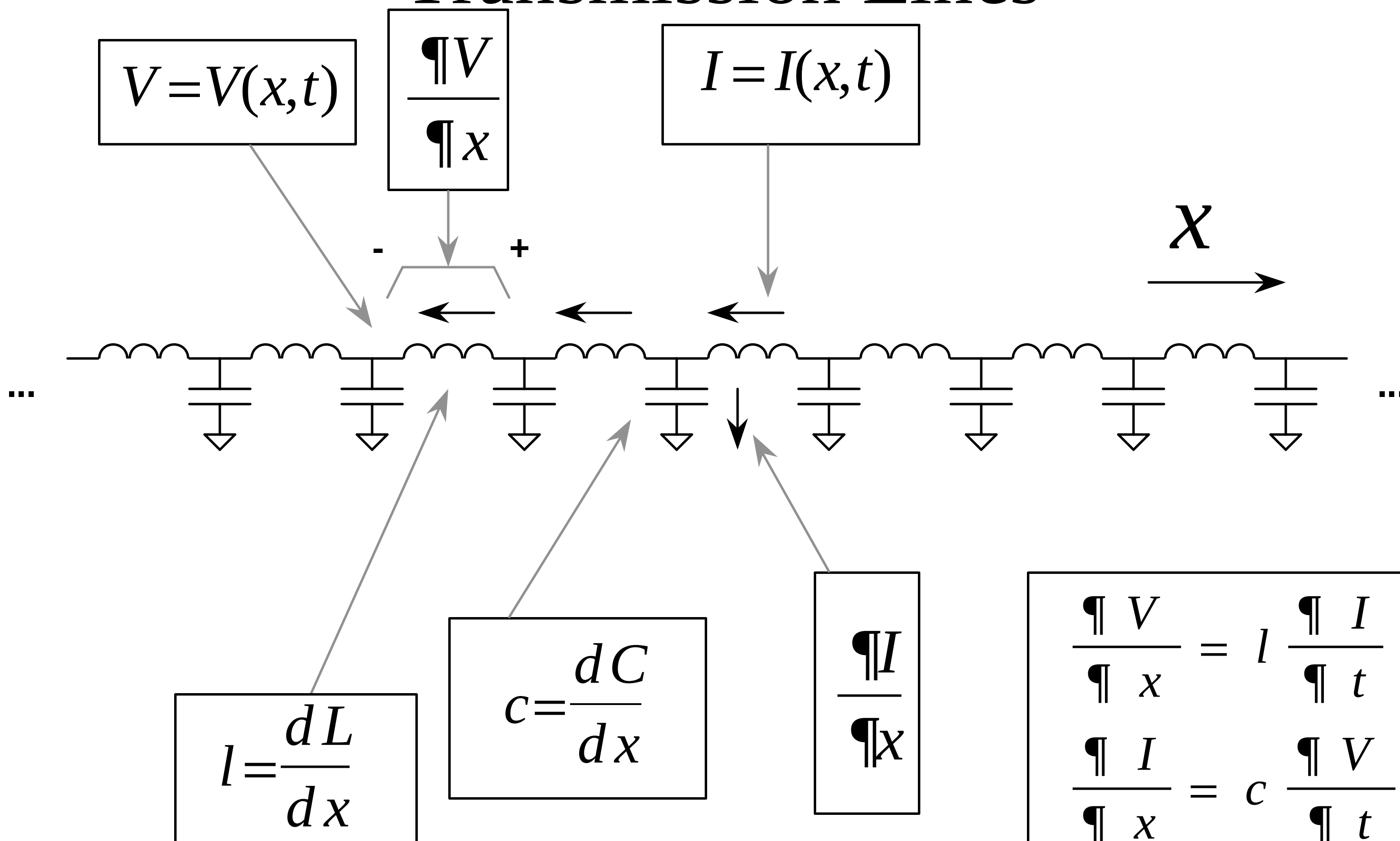
Propagating Signals must have energy

Inductance and Capacitance Stores Energy

Without termination, energy reaching the end of a transmission line has nowhere to go - so it

Echoes

Fundamental Equations of Lossless Transmission Lines



$$\frac{\partial V}{\partial x} = l \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial x} = c \frac{\partial V}{\partial t}$$

Transmission Line Math

Lets try a sinusoidal solution for V and I:

$$V = V_0\, e^{j(w_t t + w_x x)} = V_0 e^{j w_t t} e^{j w_x x}$$

$$I = I_0\, e^{j(w_t t + w_x x)} = I_0 e^{j w_t t} e^{j w_x x}$$

$$\frac{\P\ V}{\P\ x} = l\ \frac{\P\ I}{\P\ t}$$

$$\frac{\P\ I}{\P\ x} = c\ \frac{\P\ V}{\P\ t}$$

$$\longrightarrow$$

$$j w_x V_0 = l\ j w_t I_0$$

$$j w_x I_0 = c\ j w_t V_0$$

Transmission Line Algebra

$$jw_x V_0 = l \; jw_t I_0$$
$$jw_x I_0 = c \; jw_t V_0$$

$\longrightarrow$

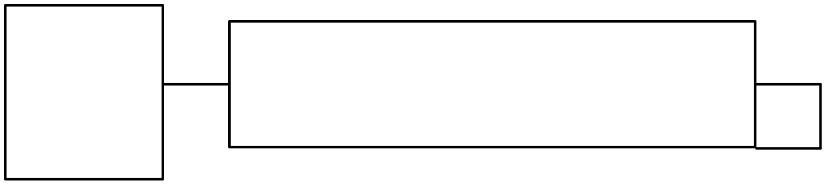
$$w_x V_0 = l \, w_t I_0$$
$$w_x I_0 = c \, w_t V_0$$

$$\frac{w_t}{w_x} = \frac{1}{\sqrt{l \, c}}$$

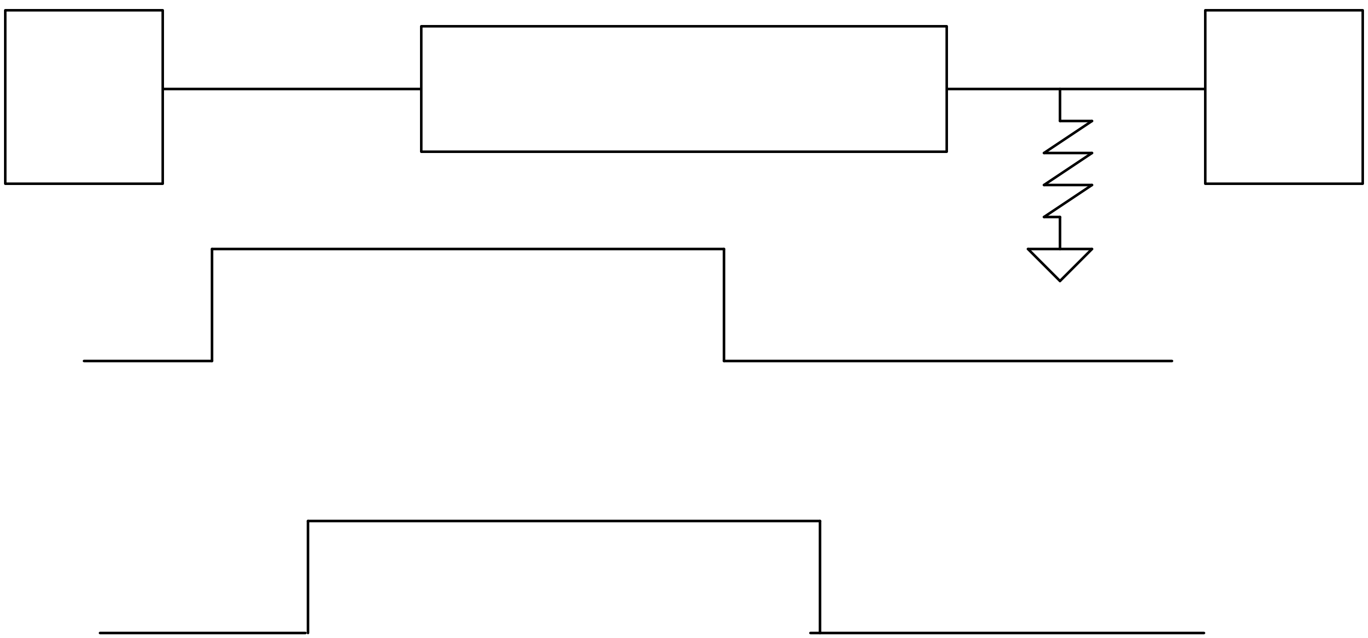
$$\frac{V_0}{I_0} = \sqrt{\frac{l}{c}}$$

The Open Transmission Line

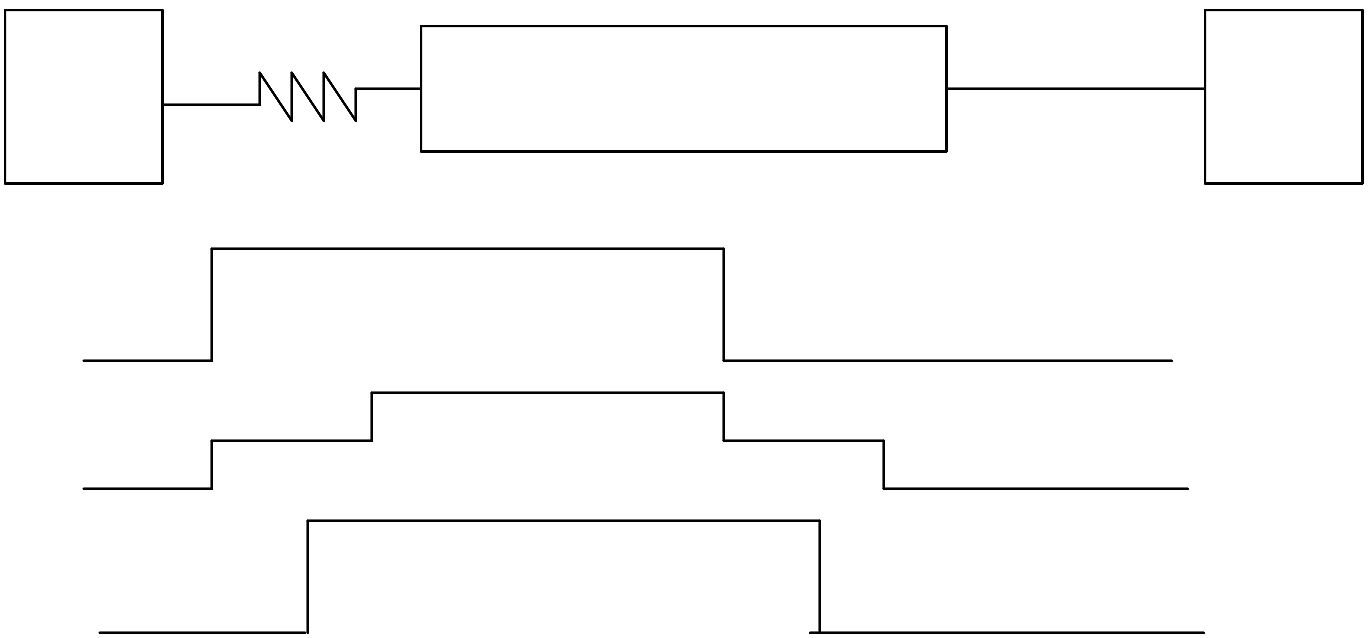
The Shorted Transmission Line



Parallel Termination



Series Termination



Series or Parallel ?

- Series:
  - No Static Power Dissipation
  - Only One Output Point
  - Slower Slew Rate if Output is Capacitively Loaded
- Parallel:
  - Static Power Dissipation
  - Many Output Points
  - Faster Slew Rate if Output is Capacitively Loaded
- Fancier Parallel Methods:
  - AC Coupled - Parallel w/o static dissipation
  - Diode Termination - “Automatic” impedance matching

When is a wire a transmission line?

$$t_{fl} = l / v$$

Rule of Thumb:

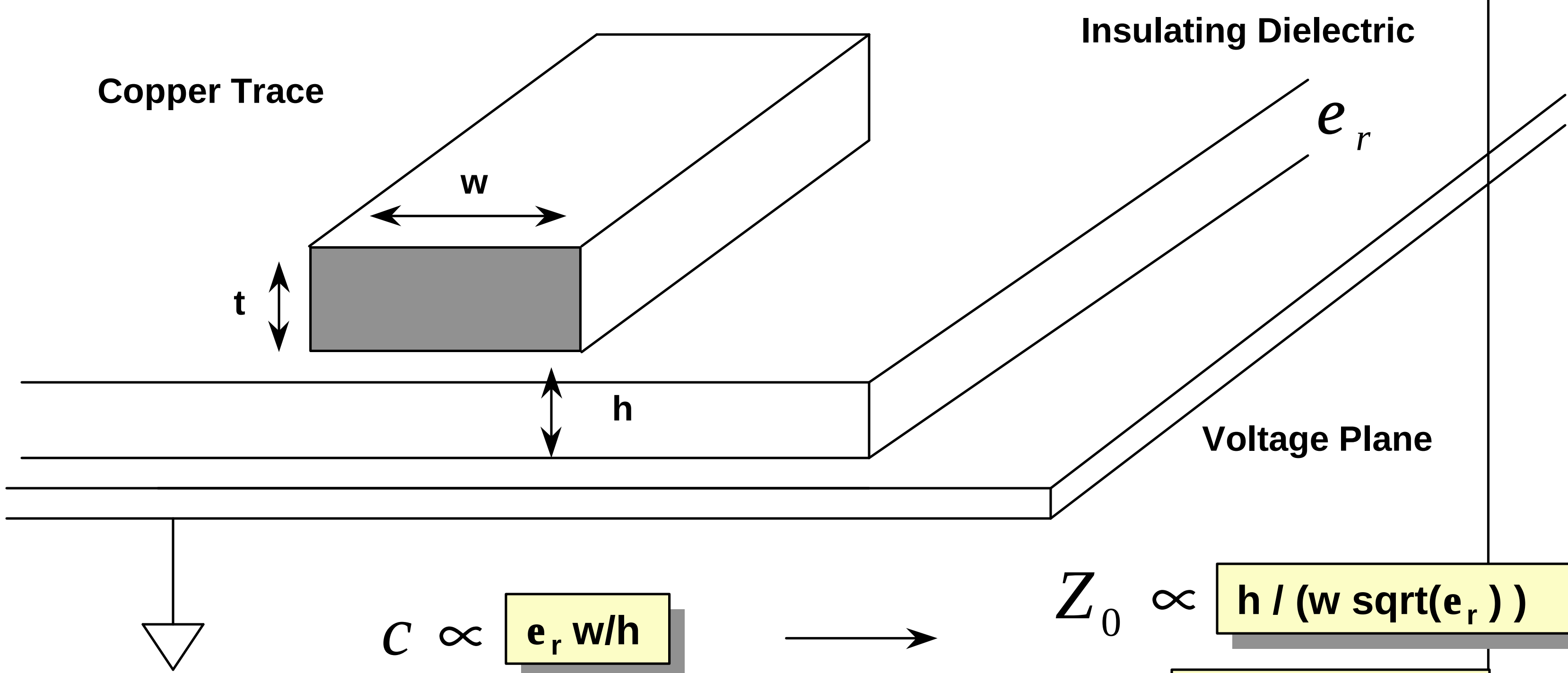
$$t_r < 2.5 t_{fl}$$

Transmission Line

$$t_r > 5 t_{fl}$$

Equipotential Line

Making Transmission Lines On Circuit Boards



$c \propto e_r \, w/h$

$l \propto h/w$

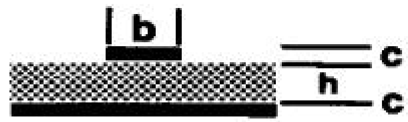
$\longrightarrow$

$Z_0 \propto h / (w \, \mathrm{sqrt}(e_r))$

$v \propto 1/\mathrm{sqrt}(e_r)$

Actual Formulas

Micro Stripline

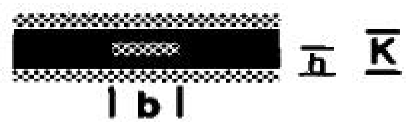


$$Z_O = \frac{87}{\sqrt{\epsilon_r + 1.41}}\, ln \left(\frac{5.98h}{0.8\,b + c} \right) \;\; \Omega$$

$$t_{PD} = 1.017\, \sqrt{0.475\,\epsilon_r + 0.67} \;\; ns/ft.$$

Stripline

c =



$$Z_O = \frac{60}{\sqrt{\epsilon_r}}\, ln \left(\frac{4K}{0.67\,\pi\, b \left(0.8 + \frac{c}{b}\right)} \right) \;\; \Omega$$

$$t_{PD} = 1.017\, \sqrt{\epsilon_r} \;\; ns/ft.$$

A Typical Circuit Board

1 Ounce Copper

G-10 Fiberglass-Epoxy

$w = 0.15 cm$

$t = 0.0038 cm$

$h = 0.038 cm$

$c = 1.9 \quad pF / cm$

$l = 2.75 \quad nH / cm$

$\longrightarrow$

$Z_0 = 38 \ \Omega$

$v = 1.4 \times 10^{10} \ cm / \sec$

$(14 \ cm / ns)$