Chapter 1 :: From Zero to One

Digital Design and Computer Architecture

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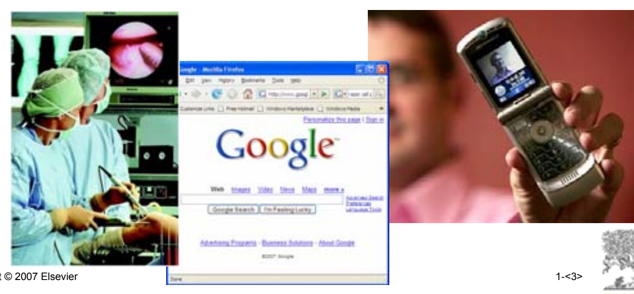
Chapter 1 :: Topics

- **Background**
- **The Game Plan**
- **The Art of Managing Complexity**
- **The Digital Abstraction**
- **Number Systems**
- **Logic Gates**
- **Logic Levels**
- **CMOS Transistors**
- **Power Consumption**

Background

- Microprocessors have revolutionized our world
  - Cell phones, Internet, rapid advances in medicine, etc.
- The semiconductor industry has grown from $21 billion in 1985 to $213 billion in 2004



The Game Plan

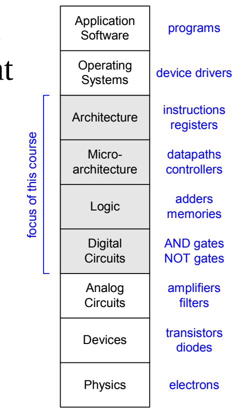
- The purpose of this course is that you:
  - Learn what's under the hood of a computer
  - Learn the principles of digital design
  - Design and build a microprocessor

The Art of Managing Complexity

- Abstraction
- Discipline
- The Three –Y's
  - Hierarchy
  - Modularity
  - Regularity

Abstraction

- Hiding details when they aren't important



Discipline

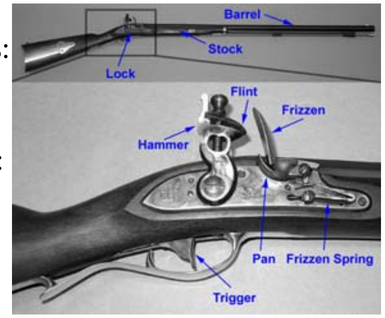
- Intentionally restricting your design choices
  - to work more productively at a higher level of abstraction
- Example: Digital discipline
  - Considering discrete voltages instead of continuous voltages used by analog circuits
  - Digital circuits are simpler to design than analog circuits – can build more sophisticated systems
  - Digital systems replacing analog predecessors:
    - I.e., digital cameras, digital television, cell phones, CDs

The Three -Y's

- Hierarchy
  - A system divided into modules and submodules
- Modularity
  - Having well-defined functions and interfaces
- Regularity
  - Encouraging uniformity, so modules can be easily reused

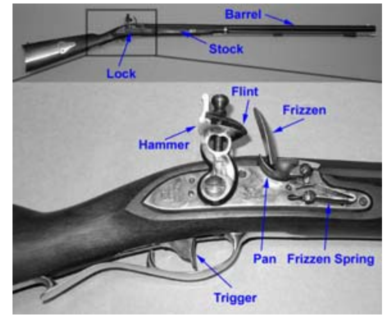
Example: Flintlock Rifle

- Hierarchy
  - Three main modules: lock, stock, and barrel
  - Submodules of lock: hammer, flint, frizzen, etc.



Example: Flintlock Rifle

- Modularity
  - Function of stock: mount barrel and lock
  - Interface of stock: length and location of mounting pins
- Regularity
  - Interchangeable parts

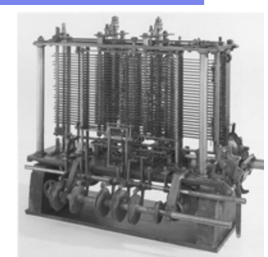


The Digital Abstraction

- Most physical variables are continuous, for example
  - Voltage on a wire
  - Frequency of an oscillation
  - Position of a mass
- Instead of considering all values, the digital abstraction considers only a discrete subset of values

The Analytical Engine

- Designed by Charles Babbage from 1834 – 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before it was finished

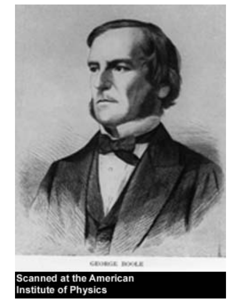


Digital Discipline: Binary Values

- Typically consider only two discrete values:
  - 1's and 0's
  - 1, TRUE, HIGH
  - 0, FALSE, LOW
- 1 and 0 can be represented by specific voltage levels, rotating gears, fluid levels, etc.
- Digital circuits usually depend on specific voltage levels to represent 1 and 0
- *Bit*: *B*inary dig*it*

George Boole, 1815 - 1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen's College in Ireland.
- Wrote *An Investigation of the Laws of Thought* (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT.



Number Systems

- Decimal numbers

1000's column
100's column
10's column
1's column

5374_{10} =

- Binary numbers

8's column
4's column
2's column
1's column

1101_2 =

Powers of Two

- 2^0 =
- 2^1 =
- 2^2 =
- 2^3 =
- 2^4 =
- 2^5 =
- 2^6 =
- 2^7 =
- 2^8 =
- 2^9 =
- 2^{10} =
- 2^{11} =
- 2^{12} =
- 2^{13} =
- 2^{14} =
- 2^{15} =

Number Conversion

- Decimal to binary conversion:
  - Convert 10101_2 to decimal
- Decimal to binary conversion:
  - Convert 47_{10} to binary

Binary Values and Range

- *N*-digit decimal number
  - Represents 10^N possible values
  - Range is: $[0, 10^N - 1]$
  - For example, a 3-digit decimal number represents 10^3 = 1000 possible values, with a range of [0, 999]
- *N*-bit binary number
  - Represents 2^N possible values
  - Range is: $[0, 2^N - 1]$
  - For example, a 3-digit binary number represents $2^3 = 8$ possible values, with a range of [0, 7] (000_2 to 111_2)

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	
9	9	
A	10	
B	11	
C	12	
D	13	
E	14	
F	15	

Hexadecimal Numbers

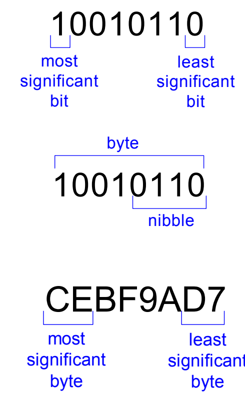
- Base 16
- Shorthand to write long binary numbers

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary
- Hexadecimal to decimal conversion:
  - Convert 0x4AF to decimal

Bits, Bytes, Nibbles…

- Bits
- Bytes & Nibbles
- Bytes



Powers of Two

- 2^{10} = 1 kilo ≈ 1000 (1024)
- 2^{20} = 1 mega ≈ 1 million (1,048,576)
- 2^{30} = 1 giga ≈ 1 billion (1,073,741,824)

Estimating Powers of Two

- What is the value of 2^{24}?
- How many values can a 32-bit variable represent?

Addition

- Decimal

```
   11  ← carries
   3734
 + 5168
 ------
   8902
```

- Binary

```
   11  ← carries
   1011
 + 0011
 ------
   1110
```


Binary Addition Examples

- Add the following 4-bit binary numbers

```
   1001
 + 0101
 ------
```

- Add the following 4-bit binary numbers

```
   1011
 + 0110
 ------
```


Overflow

- Digital systems operate on a fixed number of bits
- Addition overflows when the result is too big to fit in the available number of bits
- See previous example of 11 + 6

Signed Binary Numbers

- Sign/Magnitude Numbers
- Two's Complement Numbers

Sign/Magnitude Numbers

- 1 sign bit, N-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - Positive number: sign bit = 0
  - Negative number: sign bit = 1

$$A: \{a_{N-1}, a_{N-2}, \cdots a_2, a_1, a_0\}$$

$$A = (-1)^{a_{n-1}} \sum_{i=0}^{n-2} a_i 2^i$$

- Example, 4-bit sign/mag representations of ± 6:

  +6 =

  - 6 =

- Range of an N-bit sign/magnitude number:

Sign/Magnitude Numbers

- Problems:
  - Addition doesn't work, for example -6 + 6:

    1110
    + 0110
    10100 (wrong!)

  - Two representations of 0 (± 0):

    1000
    0000

Two's Complement Numbers

- Don't have same problems as sign/magnitude numbers:
  - Addition works
  - Single representation for 0

Two's Complement Numbers

- Same as unsigned binary, but the most significant bit (msb) has value of -2^{N-1}

$$A = a_{n-1}\left(-2^{n-1}\right) + \sum_{i=0}^{n-2} a_i 2^i$$

- Most positive 4-bit number:
- Most negative 4-bit number:
- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an N-bit two's comp number:

"Taking the Two's Complement"

- Flip the sign of a two's complement number
- Method:
  1. Invert the bits
  2. Add 1
- Example: Flip the sign of $3_{10} = 0011_2$

Two's Complement Examples

- Take the two's complement of $6_{10} = 0110_2$
- What is the decimal value of 1001_2?

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

```
   0110
+  1010
-------
```

- Add -2 + 3 using two's complement numbers

```
   1110
+  0011
-------
```


Increasing Bit Width

- A value can be extended from *N* bits to *M* bits (where *M* > *N*) by using:
  - Sign-extension
  - Zero-extension

Sign-Extension

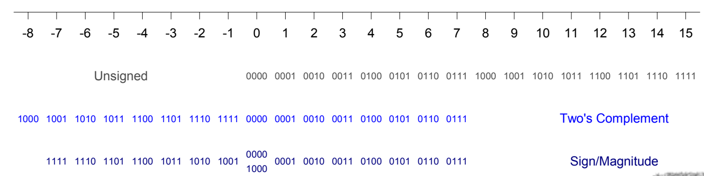
- Sign bit is copied into most significant bits.
- Number value remains the same.

- **Example 1:**
  - 4-bit representation of 3 = 0011
  - 8-bit sign-extended value: 00000011
- **Example 2:**
  - 4-bit representation of -5 = 1011
  - 8-bit sign-extended value: 11111011

Number System Comparison

Number System	Range
Unsigned	$[0, 2^N-1]$
Sign/Magnitude	$[-(2^{N-1}-1), 2^{N-1}-1]$
Two's Complement	$[-2^{N-1}, 2^{N-1}-1]$

For example, 4-bit representation:

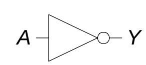


Logic Gates

- Perform logic functions:
  - inversion (NOT), AND, OR, NAND, NOR, etc.
- Single-input:
  - NOT gate, buffer
- Two-input:
  - AND, OR, XOR, NAND, NOR, XNOR
- Multiple-input

Single-Input Logic Gates

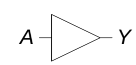
NOT



$Y = \overline{A}$

A	Y
0	
1	

BUF

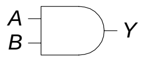


$Y = A$

A	Y
0	
1	

Two-Input Logic Gates

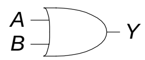
AND



$Y = AB$

A	B	Y
0	0	
0	1	
1	0	
1	1	

OR

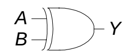


$Y = A + B$

A	B	Y
0	0	
0	1	
1	0	
1	1	

More Two-Input Logic Gates

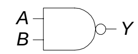
XOR



$Y = A \oplus B$

A	B	Y
0	0	
0	1	
1	0	
1	1	

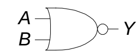
NAND



$Y = \overline{AB}$

A	B	Y
0	0	
0	1	
1	0	
1	1	

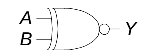
NOR



$Y = \overline{A + B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR

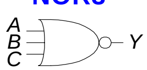


$Y = \overline{A \oplus B}$

A	B	Y
0	0	
0	1	
1	0	
1	1	

Multiple-Input Logic Gates

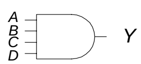
NOR3



$Y = \overline{A+B+C}$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

AND4



$Y = ABCD$

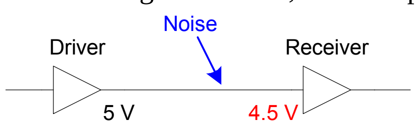
- Multi-input XOR: Odd parity

Logic Levels

- Define discrete voltages to represent 1 and 0
- For example, we could define:
  - 0 to be *ground* or 0 volts
  - 1 to be V_{DD} or 5 volts
- But what if our gate produces, for example, 4.99 volts? Is that a 0 or a 1?
- What about 3.2 volts?

Logic Levels

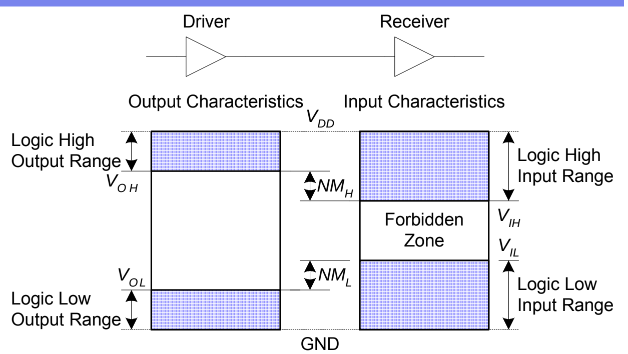
- Define a *range* of voltages to represent 1 and 0
- Define different ranges for outputs and inputs to allow for *noise* in the system
- Noise is anything that degrades the signal
- For example, a gate (driver) could output a 5 volt signal but, because of losses in the wire and other noise, the signal could arrive at the receiver with a degraded value, for example, 4.5 volts



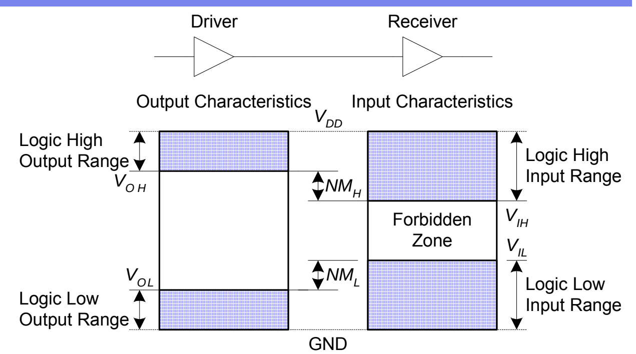
The Static Discipline

- Given logically valid inputs, every circuit element must produce logically valid outputs
- Discipline ourselves to use limited ranges of voltages to represent discrete values

Logic Levels



Noise Margins

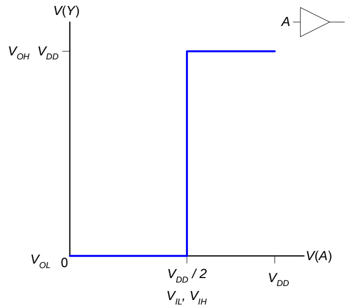


$$NM_H = V_{OH} - V_{IH}$$

$$NM_L = V_{IL} - V_{OL}$$

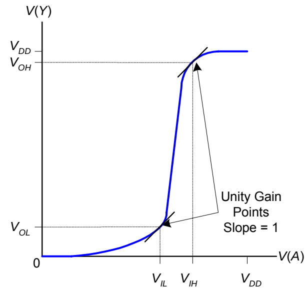
DC Transfer Characteristics

Ideal Buffer:



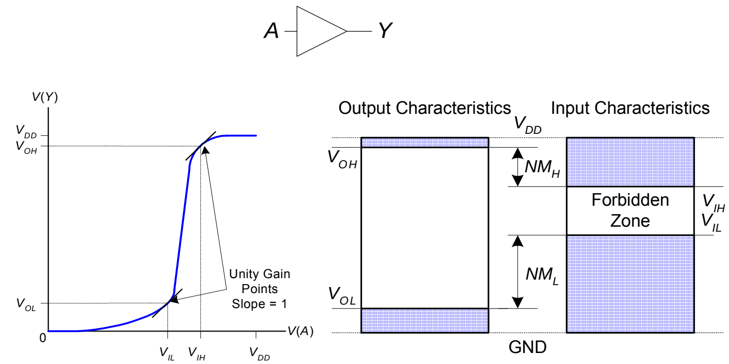
$NM_H = NM_L = V_{DD}/2$

Real Buffer:



NM_H , $NM_L < V_{DD}/2$

DC Transfer Characteristics



V_{DD} Scaling

- Chips in the 1970's and 1980's were designed using V_{DD} = 5 V
- As technology improved, V_{DD} dropped
  - Avoid frying tiny transistors
  - Save power
- 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, ..
- Be careful connecting chips with different supply voltages

Chips operate because they contain magic smoke

Proof:

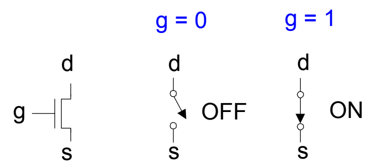
- if the magic smoke is let out, the chip stops working

Logic Family Examples

Logic Family	V_{DD}	V_{IL}	V_{IH}	V_{OL}	V_{OH}
TTL	5 (4.75 - 5.25)	0.8	2.0	0.4	2.4
CMOS	5 (4.5 - 6)	1.35	3.15	0.33	3.84
LVTTL	3.3 (3 - 3.6)	0.8	2.0	0.4	2.4
LVCMOS	3.3 (3 - 3.6)	0.9	1.8	0.36	2.7

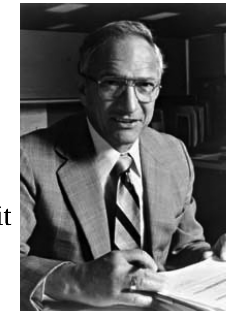
Transistors

- Logic gates are usually built out of transistors
- Transistor is a three-ported voltage-controlled switch
  - Two of the ports are connected depending on the voltage on the third port
  - For example, in the switch below the two terminals (d and s) are connected (ON) only when the third terminal (g) is 1



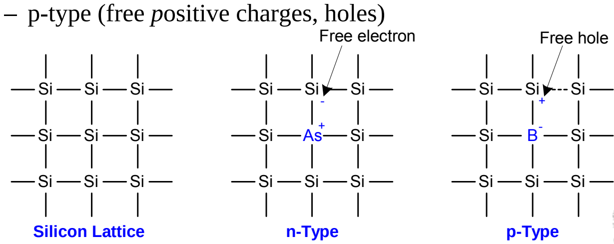
Robert Noyce, 1927 - 1990

- Nicknamed "Mayor of Silicon Valley"
- Cofounded Fairchild Semiconductor in 1957
- Cofounded Intel in 1968
- Co-invented the integrated circuit



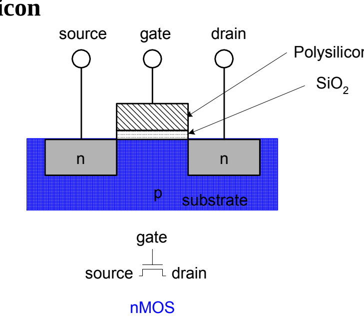
Silicon

- Transistors are built out of silicon, a semiconductor
- Pure silicon is a poor conductor (no free charges)
- Doped silicon is a good conductor (free charges)
  - n-type (free *n*egative charges, electrons)
  - p-type (free *p*ositive charges, holes)



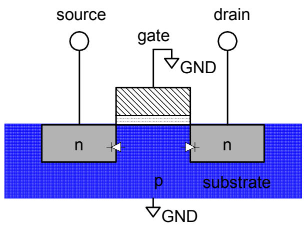
MOS Transistors

- Metal oxide silicon (MOS) transistors:
  - Polysilicon (used to be **metal**) gate
  - **Oxide** (silicon dioxide) insulator
  - Doped **silicon**

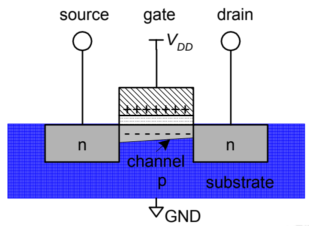


Transistors: nMOS

Gate = 0, so it is OFF (no connection between source and drain)

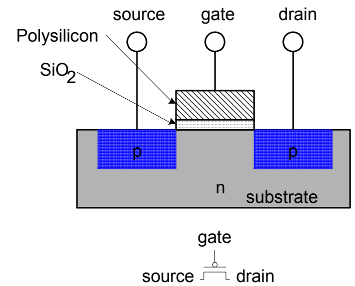


Gate = 1, so it is ON (channel between source and drain)

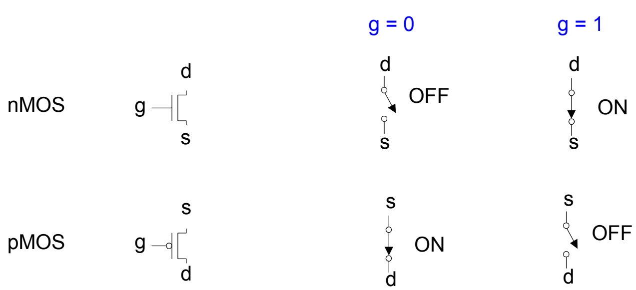


Transistors: pMOS

- pMOS transistor is just the opposite
  - ON when Gate = 0
  - OFF when Gate = 1

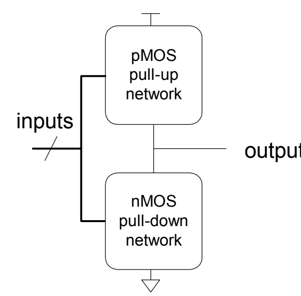


Transistor Function



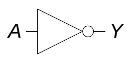
Transistor Function

- nMOS transistors pass good 0's, so connect source to GND
- pMOS transistors pass good 1's, so connect source to V_{DD}



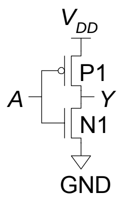
CMOS Gates: NOT Gate

NOT



$Y = \overline{A}$

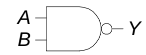
A	Y
0	1
1	0



A	P1	N1	Y
0			
1			

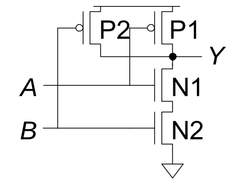
CMOS Gates: NAND Gate

NAND



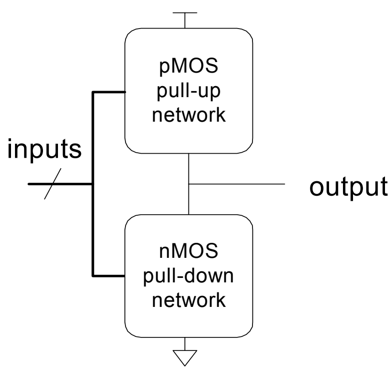
$Y = \overline{AB}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



A	B	P1	P2	N1	N2	Y
0	0					
0	1					
1	0					
1	1					

CMOS Gate Structure



NOR Gate

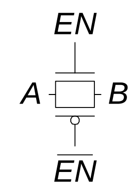
How do you build a three-input NAND gate?

Other CMOS Gates

How do you build a two-input AND gate?

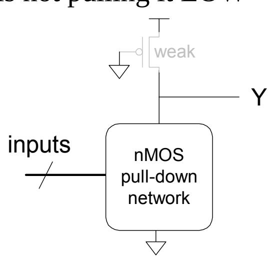
Transmission Gates

- nMOS pass 1's poorly
- pMOS pass 0's poorly
- Transmission gate is a better switch
  - passes both 0 and 1 well
- When *EN* = 1, the switch is ON:
  - *EN* = 0 and *A* is connected to *B*
- When *EN* = 0, the switch is OFF:
  - *A* is not connected to *B*



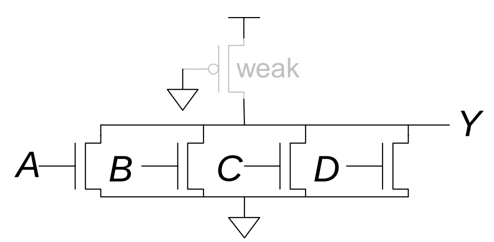
Pseudo-nMOS Gates

- nMOS gates replace the pull-up network with a *weak* pMOS transistor that is always on
- The pMOS transistor is called weak because it pulls the output HIGH only when the nMOS network is not pulling it LOW



Pseudo-nMOS Example

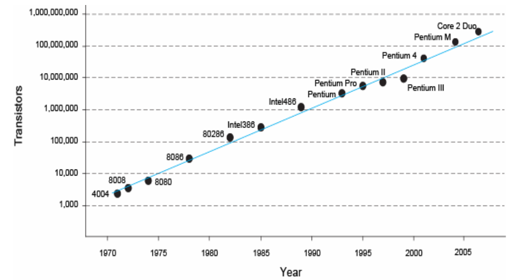
Pseudo-nMOS NOR4



Gordon Moore, 1929 -

- Cofounded Intel in 1968 with Robert Noyce.
- **Moore's Law:** the number of transistors on a computer chip doubles every year (observed in 1965)
- Since 1975, transistor counts have doubled every two years.

Moore's Law



- *"If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost $100, get one million miles to the gallon, and explode once a year . . ."*

  – Robert Cringley

Power Consumption

- Power = Energy consumed per unit time
- Two types of power consumption:
  - Dynamic power consumption
  - Static power consumption

Dynamic Power Consumption

- Power to charge transistor gate capacitances
- The energy required to charge a capacitance, C, to V_{DD} is CV_{DD}^2
- If the circuit is running at frequency f, and all transistors switch (from 1 to 0 or vice versa) at that frequency, the capacitor is charged $f/2$ times per second (discharging from 1 to 0 is free).
- Thus, the total dynamic power consumption is:

$$P_{dynamic} = \frac{1}{2} C V_{DD}^2 f$$

Static Power Consumption

- Power consumed when no gates are switching
- It is caused by the *quiescent supply current*, I_{DD}, also called the *leakage current*
- Thus, the total static power consumption is:

$$P_{static} = I_{DD}V_{DD}$$

Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - V_{DD} = 1.2 V
  - C = 20 nF
  - f = 1 GHz
  - I_{DD} = 20 mA