

## Chapter 5 :: Topics

- Introduction
- Arithmetic Circuits
- Number Systems
- Sequential Building Blocks
- Memory Arrays
- Logic Arrays

| Introduction |  |
| :---: | :---: |
| - Digital building blocks include: <br> - Gates, multiplexers, decoders, registers, arithmetic circuits, counters, memory arrays, logic arrays <br> - Building blocks are important in their own right and they demonstrate hierarchy, modularity, and regularity: <br> - They are built from a hierarchy of simpler components. <br> - They have well-defined interfaces and functions. <br> - Their regular structure is easily extended to different sizes. <br> - We'll use many of these building blocks to build a microprocessor in Chapter 7 |  |
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## Multibit Adder, also called CPA

- Several types of carry propagate adders (CPAs) are:
- Ripple-carry adders
(slow)
- Carry-lookahead adders
(fast)
- Prefix adders
(faster)
- Carry-lookahead and prefix adders are faster for large adders but require more hardware.


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## Ripple-Carry Adder

- Chain 1-bit adders together
- Carry ripples through entire chain
- Disadvantage: slow


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## Carry-Lookahead Adder

- Computes the carry out ( $C_{\text {out }}$ ) for $N$-bit blocks first, so the carry doesn't have to ripple through the entire chain.
- Does this by computing generate $(G)$ and propagate $(P)$ signals for columns and then $N$-bit blocks.
- A column (bit $i$ ) can produce a carry out by either generating a carry out or propagating a carry in to the carry out.
- We define generate $\left(G_{i}\right)$ and propagate $\left(P_{i}\right)$ signals for each column:
- A column will generate a carry out if $A_{i}$ AND $B_{i}$ are both 1 .

$$
G_{i}=A_{i} B_{i}
$$

- A column will propagate a carry in to the carry out if $A_{i}$ OR $B_{i}$ is 1 .

$$
P_{i}=A_{i}+B_{i}
$$

- We compute the carry out of a column $\left(C_{i}\right)$ as: copyight e2007 Esesever $\quad C_{i}=A_{i} B_{i}+\left(A_{i}+B_{i}\right) C_{i-1}=G_{i}+P_{i} C_{i-1}$
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## Carry-Lookahead Adder

- Now we compute generate and propagate signals for $N$-bit blocks.
- For example, we can calculate generate and propagate signals for a 4-bit block ( $G_{3: 0}$ and $P_{3: 0}$ ) :
- A 4-bit block will generate a carry out if column 3 generates a carry $\left(G_{3}\right)$ or if column 3 propagates a carry $\left(P_{3}\right)$ that was generated or propagated in a previous column as described by the following equation:

$$
G_{3: 0}=G_{3}+P_{3}\left(G_{2}+P_{2}\left(G_{1}+P_{1} G_{0}\right)\right.
$$

- A 4-bit block will propagate a carry in to the carry out if all of the columns propagate the carry:

$$
P_{3: 0}=P_{3} P_{2} P_{1} P_{0}
$$

- We compute the carry out of the 4 -bit block $\left(C_{i}\right)$ as:

$$
C_{i}=G_{i, j}+P_{i, j} C_{i-1}
$$

$$
5 \ll 11>
$$




## Prefix Adder

- Computes generate and propagate signals for all of the columns to perform addition even faster.
- Computes $G$ and $P$ for 2-bit blocks, then 4-bit blocks, then 8 -bit blocks, etc. until the generate and propagate signals are known for each column.
- Thus, the prefix adder has $\log _{2} N$ stages.
- The strategy is to compute the carry in $\left(C_{i-1}\right)$ for each of the columns as fast as possible and then to compute the sum:

$$
S_{i}=\left(A_{i} \oplus B_{i}\right) \oplus C_{i-1}
$$

- The delay of an $N$-bit carry-lookahead adder is generally much faster than a ripple-carry adder for $N>16$

[^0]
## Carry-Lookahead Adder Delay

- The delay of an $N$-bit carry-lookahead adder with $k$-bit blocks is:

$$
t_{C L A}=t_{p g}+t_{p g \_ \text {block }}+(N / k-1) t_{\mathrm{AND}_{2} \mathrm{OR}}+k t_{F A}
$$

where

- $t_{p g}$ is the delay of the column generate and propagate gates
- $t_{p g \_ \text {block }}$ is the delay of the block generate and propagate gates
- $t_{\text {AND_or }}$ is the delay from $C_{\text {in }}$ to $C_{\text {out }}$ of the final AND/OR gate in the $k$-bit CLA block


## Prefix Adder

- A carry is generated by being either generated in a column or propagated from a previous column.
- Define column -1 to hold $C_{\text {in }}$, so

$$
G_{-1}=C_{\text {in }}, P_{-1}=0
$$

- Then,

$$
C_{i-1}=G_{i-1:-1}
$$

because there will be a carry out of column $i-1$ if the block spanning columns $i-1$ through -1 generates a carry.

- Thus, we can rewrite the sum equation as:

$$
S_{i}=\left(A_{i} \oplus B_{i}\right) \oplus G_{i: 1:-1}=P_{i} \oplus G_{i: 1:-1}
$$

- Goal:
- Quickly compute $\mathrm{G}_{0:-1}, \mathrm{G}_{1:-1}, \mathrm{G}_{2:-1}, \mathrm{G}_{3:-1}, \mathrm{G}_{4:-1}, \mathrm{G}_{5:-1}, \ldots$ Copyight© 2007 Esseverer


## Prefix Adder

- The generate and propagate signals for a block spanning bits $i: j$ are:

$$
\begin{aligned}
& G_{i: j}=G_{i: k}+P_{i: k} G_{k-1 ; j} \\
& P_{\mathrm{i}: \mathrm{j}}=P_{i: k} P_{k-1: \mathrm{j}}
\end{aligned}
$$

- In words, these prefixes describe that:
- A block will generate a carry if the upper part (i:k) generates a carry or of the upper part propagates a carry generated in the lower part ( $k-1: j$ )
- A block will propagate a carry if both the upper and lower parts propagate the carry.


## Prefix Adder Delay

- The delay of an $N$-bit prefix adder is:

$$
t_{P A}=t_{p g}+\log _{2} N\left(t_{p g \_ \text {prefix }}\right)+t_{\mathrm{XOR}}
$$

where

- $t_{p g}$ is the delay of the column generate and propagate gates
- $t_{p g \text { _refix }}$ is the delay of the black prefix cell (AND-OR gate)




## Comparator: Less Than

- For unsigned numbers





## Set Less Than (SLT) Example



- Configure a 32-bit ALU for the set if less than (SLT) operation. Suppose $A=25$ and $B=32$.
- Because $A$ is indeed less than $B$, we expect Y to be the 32-bit representation of 1 (0x00000001).
- For SLT, $F_{2 \cdot 0}=111$
- $F_{2}=1$ configures the adder unit as a subtracter. So 25-32 = -7.
- The two's complement representation of -7 has a 1 in the most significant bit, so $S_{31}=1$.
_ With $F_{10}=11$, the final multiplexer selects $Y=S_{31}=1$


## Shifters

- Logical shifter: shifts value to left or right and fills empty spaces with 0's
- Ex: $11001 \gg 2$ =
- Ex: $11001 \ll 2=$
- Arithmetic shifter: same as logical shifter, but on right shift, fills empty spaces with the old most significant bit (msb).
- Ex: $11001 \ggg 2$ =
- Ex: $11001 \lll 2$ =
- Rotator: rotates bits in a circle, such that bits shifted off one end are shifted into the other end
- Ex: 11001 ROR $2=$
- Ex: 11001 ROL 2 =

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## Number Systems

- What kind of numbers do you know how to represent using binary representations?
- Positive numbers
- Unsigned binary
- Negative numbers
- Two's complement
- Sign/magnitude numbers
- What about fractions?



## Fixed-Point Numbers

- Fixed-point representation of 6.75 using 4 integer bits and 4 fraction bits:

$$
\begin{aligned}
& 01101100 \\
& 0110.1100 \\
& 2^{2}+2^{1}+2^{-1}+2^{-2}=6.75
\end{aligned}
$$

- The binary point is not a part of the representation but is implied.
- The number of integer and fraction bits must be agreed upon by those generating and those reading the number. Copyight © 2007 Elsevier
$\qquad$


## Signed Fixed-Point Numbers

- As with integers, negative fractional numbers can be represented two ways:
- Sign/magnitude notation
- Two's complement notation
- Represent $-6.5_{10}$ using an 8-bit binary representation with 4 integer bits and 4 fraction bits.
- Sign/magnitude:
- Two's complement

1. +6.5 :
2. Invert bits:
3. Add 1 ulp: $\qquad$

## Floating-Point Numbers

- The binary point floats to the right of the most significant 1.
- Similar to decimal scientific notation.
- For example, write $273_{10}$ in scientific notation:
- Move the decimal point to the left of the most significant digit and increase the exponent:

$$
273=2.73 \times 10^{2}
$$

- In general, a number is written in scientific notation as:

$$
\pm \mathrm{M} \times \mathrm{B}^{\mathrm{E}}
$$

Where,

- M = mantissa
- $\mathrm{B}=$ base
- $\mathrm{E}=$ exponent
- In the example, $\mathrm{M}=2.73, \mathrm{~B}=10$, and $\mathrm{E}=2$

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## Floating-Point Numbers

- We represent floating-point numbers using 32 bits: 1 sign bit, 8 exponent bits, and the remaining 23 bits for the mantissa.

| bit | 8 bits | 23 bits |
| :--- | :--- | :--- |
|  |  |  |
| Sign | Exponent | Mantissa |

- Example: represent the value $228_{10}$ using a 32 -bit floating point representation.
- The following slides show three versions of floating-point representation for $228_{10}$.
- The final version is called the IEEE 754 floating-point standard.
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## Floating-Point Representation 2

- You may have noticed that the first bit of the mantissa is always 1 , since the binary point floats to the right of the most significant 1 :
$-228_{10}=11100100_{2}=1.11001 \times 2^{7}$
- Thus, storing the most significant 1 , also called the implicit leading 1 , is redundant information.
- We can store just the fraction bits in the 23-bit field. The leading 1 is implied.



## Floating-Point Representation 3

- The final change is to store a biased exponent. The IEEE 754 standard uses a bias of 127 .
- Biased exponent $=$ bias + exponent
- For example, an exponent of 7 would be stored as:

$$
127+7=134=0 \times 10000110_{2}
$$

- Thus, the IEEE 754 32-bit floating-point representation of $228_{10}$ is:



## Floating-Point Example

- Write the value $-58.25_{10}$ using the IEEE 754 32-bit floatingpoint standard.
- First, convert the decimal number to binary:

$$
-58.25_{10}=
$$

- Next, fill in each field in the 32 -bit number:
- The sign bit is
- The 8 exponent bits
- The remaining 23 bits are the fraction bits.

- Written in hexadecimal, this 32-bit value is: Copyight © 2007 Elsevier


## Floating-Point Number Precision

- Single-Precision:
- 32-bit notation
- 1 sign bit, 8 exponent bits, 23 fraction bits
- bias $=127$
- Double-Precision:
- 64-bit notation
- 1 sign bit, 11 exponent bits, 52 fraction bits
- bias $=1023$



## Floating-Point Addition

1. Extract exponent and fraction bits
2. Prepend leading 1 to form mantissa


## Floating-Point Addition: Example

1. Extract exponent and fraction bits

| 1 bit | 8 bits | 23 bits |
| :---: | :---: | :---: |
| 0 | 01111111 | 10000000000000000000000 |
| Sign | Exponent | Fraction |
| 1 bit | 8 bits | 23 bits |
| 0 | 10000000 | 1010000000000000000000 |
| Sign | Exponent | Fraction |

For first number (N1): $\quad \mathrm{S}=0, \mathrm{E}=127, \mathrm{~F}=.1$
For second number (N2): $\quad S=0, E=128, F=.101$
2. Prepend leading 1 to form mantissa

N1: 1.1
$\mathrm{N} 2: \quad 1.101$

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## Floating-Point Addition: Example

6. Normalize mantissa and adjust exponent if necessary $10.011 \times 2^{1}=1.0011 \times 2^{2}$
7. Round result

No need (fits in 23 bits)
8. Assemble exponent and fraction back into floating-point format
$\mathrm{S}=0, \mathrm{E}=2+127=129=10000001_{2}, \mathrm{~F}=001100$.

| 1 bit | 8 bits 23 bits |  |
| :---: | :---: | :---: |
| 0 | 10000001 | 00110000000000000000000 |
| Sign | Exponent | Fraction |

Written in hexadecimal: 0x40980000

## Shift Register

- Shift a new value in on each clock edge
- Shift a value out on each clock edge
- Serial-to-parallel converter: converts serial input $\left(S_{\text {in }}\right)$ to parallel output ( $Q_{0: N-1}$ )
- Digital clock display
- Program counter: used in computers to keep track of the current instruction that is executing


Symbol:


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Implementation:


## Shift Register with Parallel Load

- When Load $=1$, acts as a normal $N$-bit register
- When Load $=0$, acts as a shift register
- Now can act as a serial-to-parallel converter $\left(\mathrm{S}_{\mathrm{in}}\right.$ to $\left.Q_{0: \mathrm{N}-1}\right)$ or a parallel-to-serial converter ( $D_{0: N-1}$ to $S_{\text {out }}$ )



## Memory Arrays

- Memory arrays efficiently store large amounts of data.
- Three common types of memory arrays:
- Dynamic random access memory (DRAM)
- Static random access memory (SRAM)
- Read only memory (ROM)
- An $M$-bit data value can be read or written at each unique $N$ bit address.



## Memory Array: Example

- The memory array below is a $2^{2} \times 3$-bit array.
- The word size is 3-bits.
- For example, the 3-bit word stored at address 10 is 100 .

An array with $N$ address bits and $M$ data bits has $2^{N}$ rows and

$$
\text { s } 10 .
$$

Example:

- Width: number of columns in a memory array (the word size)
- Array size is given as depth $\times$ width


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## Types of Memory

- Random access memory (RAM): volatile
- Read only memory (ROM): nonvolatile



## RAM

- Random access memory
- Volatile: loses its data when the power is turned off
- Can be read and written quickly
- Main memory in your computer is RAM (specifically, DRAM)
- Historically called random access memory because any data word can be accessed as easily as any other (in contrast to sequential access memories such as a tape recorder).

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## Robert Dennard, 1932 -

- Invented DRAM in 1966 at IBM
- Others were skeptical that the idea would work
- By the mid-1970’s DRAM was in virtually all computers





## Fujio Masuoka, 1944-

- Developed memories and high speed circuits at Toshiba from 1971-1994
- Invented Flash memory as an unauthorized project pursued during nights and weekends in the late 1970's.
- The process of erasing the memory reminded him of the flash of a camera
- Toshiba slow to commercialize the idea; Intel was first to market in 1988
- Flash has grown into a $\$ 25$ billion per year market.


[^1]

## Example: Logic with ROMs

- Implement the following logic functions using a $2^{2} \times 3$-bit ROM:
- $X=A B$
- $Y=A+B$
- $Z=A \bar{B}$


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## Logic with Memory Arrays

| $\begin{gathered} 2: 4 \\ \text { Decoder } \end{gathered}$ | wordline | bitline ${ }_{2}$ |  |  | bititine |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |
| 10 |  | stored | $\begin{aligned} & \text { stored } \\ & \text { bit }=1 \end{aligned}$ | $\begin{array}{\|l} \text { stored } \\ \text { bit }=0 \end{array}$ |  |
|  | wordine ${ }_{2}$ |  |  |  |  |
|  |  | stored | stored | stored |  |
| 01 | wordine ${ }_{1}$ | bit $=1$ | bit $=0$ | bit $=0$ |  |
|  |  | stored | stored | stored |  |
|  | wordine | bit $=1$ | bit $=1$ | bit $=0$ |  |
| 00 |  |  |  | stored |  |
|  |  | bit $=0$ | bit $=1$ | bit $=1$ |  |

$$
\operatorname{Data}_{1}=\overline{A_{1}}+A_{0}
$$

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$$
\operatorname{Data}_{2}=A_{1} \oplus A_{0}
$$

$$
\operatorname{Data}_{0}=\overline{A_{1}} \overline{A_{0}}
$$

## Logic with Memory Arrays

- Memory arrays used to perform logic are called lookup tables (LUTs).
- The user looks up the value of the output at each input combination (address).


## Logic with Memory Arrays

- Implement the following logic functions using a $2^{2} \times 3$-bit memory array:
- $X=A B$

$$
\begin{aligned}
& \begin{array}{l}
-Y=A+B \\
-Z=A \bar{B}
\end{array}
\end{aligned}
$$

- 



## Multi-ported Memories

- Port: address/data pair
- 3-ported memory
- 2 read ports (A1/RD1, A2/RD2)
- 1 write port (A3/WD3, WE3 enables writing)
- Small multi-ported memories are called register files


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## Logic Arrays

- Programmable logic arrays (PLAs)


## PLAs

- $\mathrm{X}=\mathrm{ABC}+\mathrm{ABC}$
- AND array followed by OR array
- Perform combinational logic only
- Fixed internal connections
- Field programmable gate arrays (FPGAs)
- Array of configurable logic blocks (CLBs)
- Perform combinational and sequential logic
- Programmable internal connections




## FPGAs

- Composed of:
- CLBs (Configurable logic blocks): to perform logic
- IOBs (Input/output buffers): to interface with outside world
- Programmable interconnection: to connect CLBs and IOBs
- Some FPGAs include other building blocks such as multipliers and RAMs




## Xilinx Spartan CLB

- The Spartan CLB has:
- 3 LUTs:
- F-LUT ( $2^{4}$ x 1-bit LUT)
- G-LUT ( $2^{4}$ x 1 -bit LUT)
- H-LUT ( $2^{3}$ x 1 -bit LUT)
-2 registered outputs:
- $X Q$
- $Y Q$
- 2 combinational outputs.
- $X$
- Y


## CLB Configuration Example

- Show how to configure the Spartan CLB to perform the following functions:
- $X=\overline{A B C}+A B \bar{C}$
- $Y=A \bar{B}$

|  | (A) | (B) | (C) ${ }^{(X)}$ |  | (A) (B) ${ }^{(Y)}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F4 | F3 | F2 | F1 | F | G4 | G3 | G2 | G1 | G |
| X | 0 | 0 | 0 | 0 | X | X | 0 | 0 | 0 |
| x | 0 | 0 | 1 | 1 | X | x | 0 | 1 | 0 |
| x | 0 | 1 | 0 | 0 | X | x | 1 | 0 | 1 |
| X | 0 | 1 | 1 | 0 | X | X | 1 | 1 | 0 |



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## FPGA Design Flow

- A CAD tool (such as Xilinx Project Navigator) is used to design and implement a digital system.
- The user enters the design using schematic entry or an HDL.
- The user simulates the design.
- A synthesis tool converts the code into hardware and maps it onto the FPGA.
- The user uses the CAD tool to download the configuration onto the FPGA
- This configures the CLBs and the connections between them and the IOBs.

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