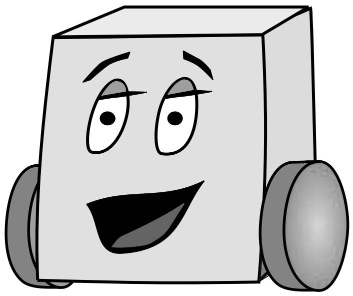


E11: Autonomous Vehicles

Fall 2011

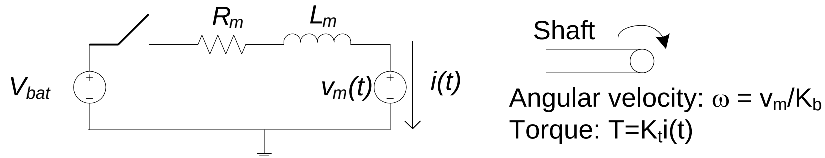
Harris & Harris

Problem Set 8: Motors

1. Motor Transient Analysis and Torque

In the figure below, a battery with voltage V_{bat} is connected to a motor and gearbox through a switch. The motor coil has a resistance R_m and inductance L_m. Ben Bitdiddle clamps the wheel so it cannot spin ($\omega = 0$). He closes the switch at time $t = 0$. The parameters for the motor attached to the gear box (with a 114.7:1 gear ratio) are:

$R_m = 7.56\ \Omega$
$L_m = 2.45$ mH
$K_b = 0.56$ V / (rad/s)
$K_t = 0.178$ N-m / A



a) Write a differential equation describing $i(t)$ valid for $t > 0$.

b) Compute an expression for $i(t)$. (Hint: guess $i(t)$ is in the form of a constant plus a decaying exponential.)

c) Plot $i(t)$ from 0 to 1 ms.

d) How much torque does the motor exert in the instant after the switch closes?

e) How much stall torque does the motor exert in steady state?

2. On your mark, get set, go!

The equation of motion for the motor (including the gear train) is

$$J\frac{d\omega(t)}{dt} + b\omega(t) = T(t)$$

The wheel can be approximated as a thick-walled cylinder with inner radius r_1, outer radius r_2, and mass m. Its moment of inertia is

$$J_w = m\frac{r_1^2 + r_2^2}{2}$$

The bot itself, with mass M, contributes additional rotational inertia. Its moment is

$$J_{bot} = Mr_2^2$$

The total moment of inertia is $J = J_w + J_{bot}$.

Use the same motor parameters from the previous problem. Assume that the mechanical motion is sufficiently slower than the electrical response of the coil that the inductance L_m can be ignored (i.e. torque abruptly steps from 0 to the steady state value when the switch is closed).

a) When V_{bat} = 8 V is applied, the wheel shaft spins freely at a steady state angular velocity of 125 rpm and draws 82 mA. Compute the friction coefficient, b.

b) The mass of the wheel is concentrated along the tire with an outer *diameter* of 2.25", an inner *diameter* of 1.75", and a mass of 23.6 g. The robot as a whole has a mass of 418 g. Compute the total moment of inertia of the system.

c) At time 0, Ben closes the switch to connect the battery and the robot starts rolling. How long does it take to travel the first foot? (Hint: you can predict a form for the solution of the differential equation based on your physical understanding of how the robot should behave, and solve for angular velocity. With some algebra and integration, you can calculate the linear velocity and distance traveled as a function of time. This function can't be inverted in closed form. However, by guessing and checking, you can determine the time to cover the first foot.)

d) By the time the robot has traveled the first foot, you should observe that it has essentially reached the constant angular velocity given in part (a). This is called *steady state* operation. How long does it take to travel each subsequent foot? (Hint: this can be answered using just algebra, even if you didn't solve part (c).)