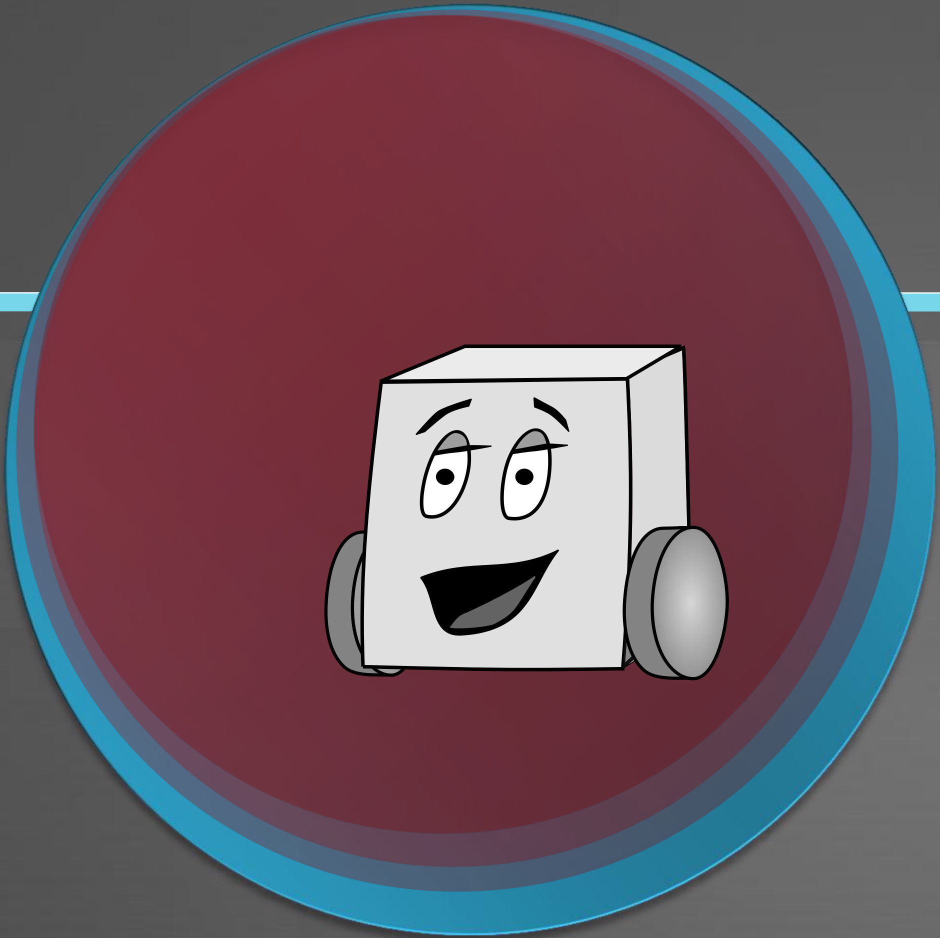

E11 Lecture 16: Mechanical Performance

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Fall 2011

Rules Clarification

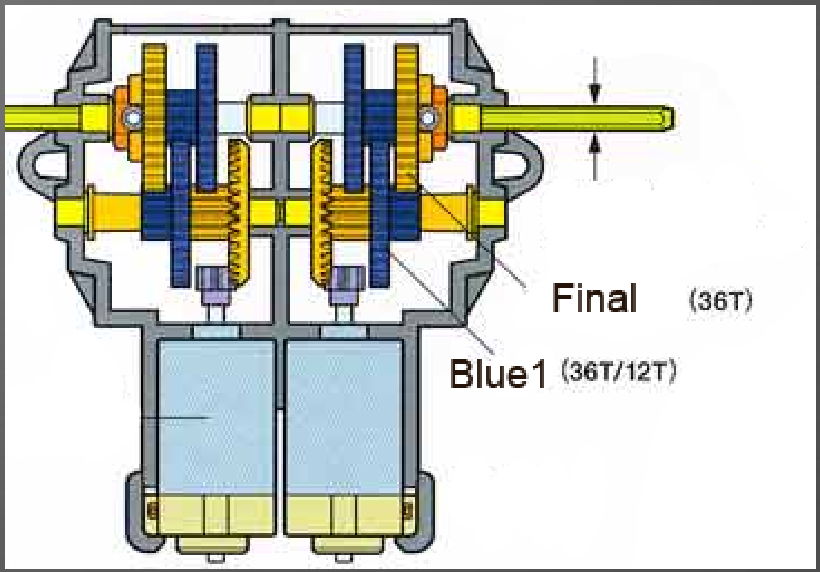
- Chassis must fit in 7" sphere
- Other parts can hang off
  - PCB
  - Wheels
  - Sensors
  - …

Outline

- Motor Modeling
- System Characterization
- Motor Performance
  - How fast can the robot go?
  - How hard can it push?

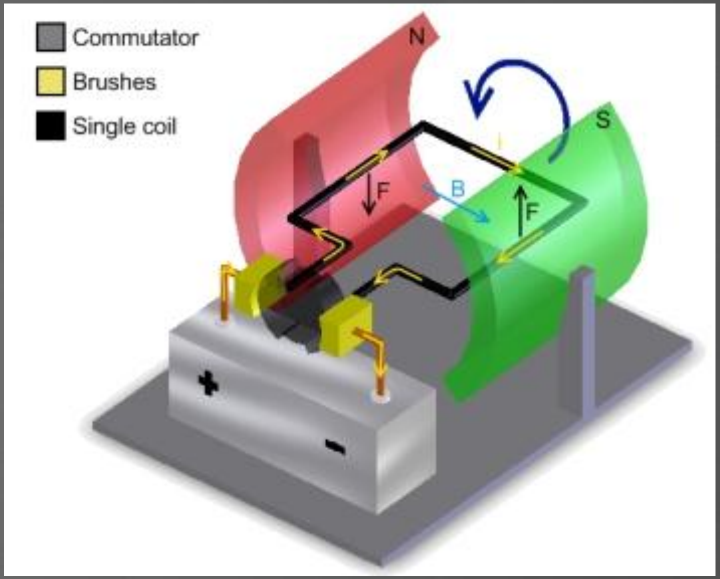
Mechanical Subsystem

- Pololu 1117 DC Brush Motor
  - 11500 RPM @ 6V
  - 70 mA free-run current
  - 800 mA stall current
    - When shaft is held still
- 114.7:1 Gear Train
- 2.25" diameter wheels



DC Motor Review

- Stator has a permanent magnet
- Rotor contains an electromagnet
  - a.k.a. armature
- Commutator reverses the current in the rotor to make motor spin
- Electromagnet coil is a wire with
  - Resistance
  - Inductance

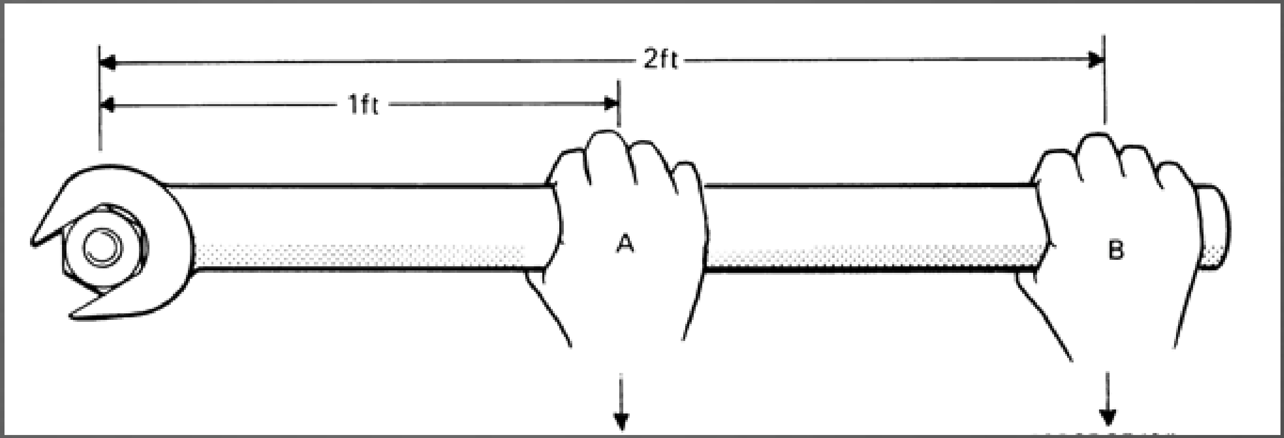


http://humanoids.dem.ist.utl.pt/servo/overview.html

Force at a Distance

- How does the force required to tighten the bolt compare at point A and point B?

A. $F_A > F_B$
B. $F_B > F_A$
C. $F_B = F_A$



http://mdmetric.com/tech/torqcht2.htm

Torque

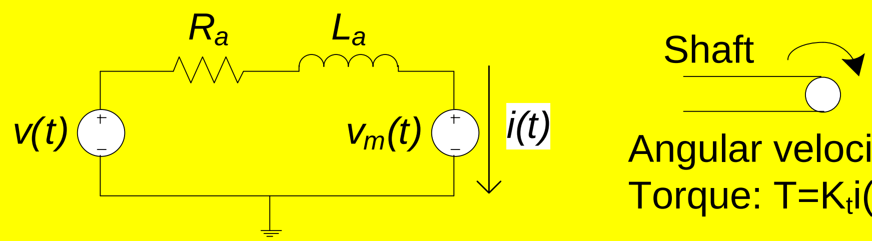
- Torque can be thought of as how hard a system is turning
- A force applied at a greater distance creates more torque
  - $T = F \times d$
  - Units of Newton-meters (N-m)

Motor Modeling

- Coupled electro-mechanical system
  - Electrical
    - Rotor/armature coil has
      - R_a: resistance
      - L_a: inductance
      - V_m: back electromotive force (EMF)
    - T: Torque is proportional to current i
  - Mechanical
    - Shaft and wheel have
      - J_m: moment of inertia
      - b: damping coefficient
    - V_m is proportional to angular velocity ω
- Relate angular velocity and torque to input voltage v

Electromechanical System

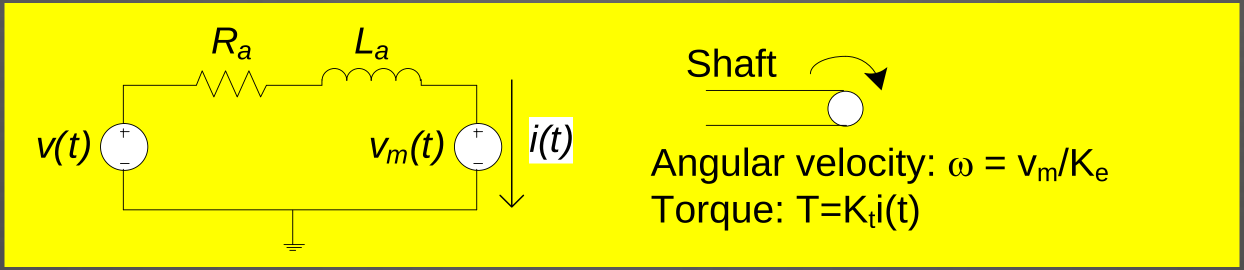
- Electrical
  - Rotor coil has
    - R_a: resistance
    - L_a: inductance
    - V_m: back electromotive force (EMF)
  - T: Torque is proportional to i
- Mechanical
  - Shaft and wheel have
    - J_m: moment of inertia
    - b: damping coefficient
  - ω: angular velocity is proportional to V_m



Angular velocity: $\omega = v_m/K_e$
Torque: $T=K_t i(t)$

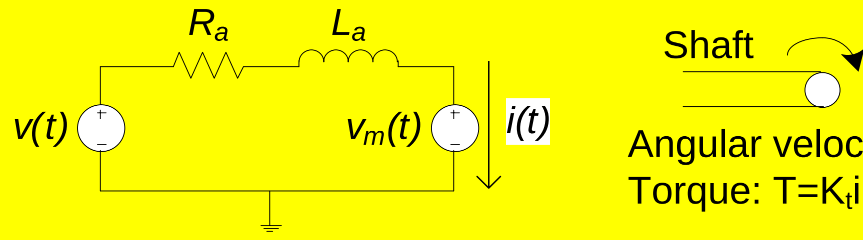
Electrical Governing Equation

$$v(t) = R_a i(t) + L_a \frac{di(t)}{dt} + v_m(t)$$



Mechanical Equation of Motion

$$J_m \frac{d\omega(t)}{dt} + b\omega(t) = T(t)$$



Angular velocity: $\omega = v_m/K_e$
Torque: $T=K_t i(t)$

Coupled Equation

$v(t) = R_a i(t) + L_a \dfrac{di(t)}{dt} + v_m(t)$	(1)	Electrical Model
$v_m(t) = K_e \omega(t)$	(2)	Back EMF
$T(t) = K_t i(t)$	(3)	Torque
$T(t) = J_m \dot{\omega}(t) + b\omega(t)$	(4)	Mechanical Model
$v(t) = \dfrac{L_a}{K_t}\dot{T}(t) + \dfrac{R_a}{K_t}T(t) + K_e \omega(t)$	(5)	Substitute (2) and (3) in (1)
$v(t) = \dfrac{L_a J_m}{K_t}\ddot{\omega}(t) + \left(\dfrac{L_a b + R_a J_m}{K_t}\right)\dot{\omega}(t) + \left(\dfrac{R_a b + K_t K_e}{K_t}\right)\omega(t)$	(6)	Substitute (4) in (5)

Coupled Equation

$$v(t) = \frac{L_a J_m}{K_t}\ddot{\omega}(t) + \left(\frac{L_a b + R_a J_m}{K_t}\right)\dot{\omega}(t) + \left(\frac{R_a b + K_t K_e}{K_t}\right)\omega(t)$$

- 2nd order linear differential equation relating ω(t) to v(t)
  - Speed-voltage relationship
- You'll learn how to solve these in DiffEq
  - Solution involves decaying exponentials and sinusoids

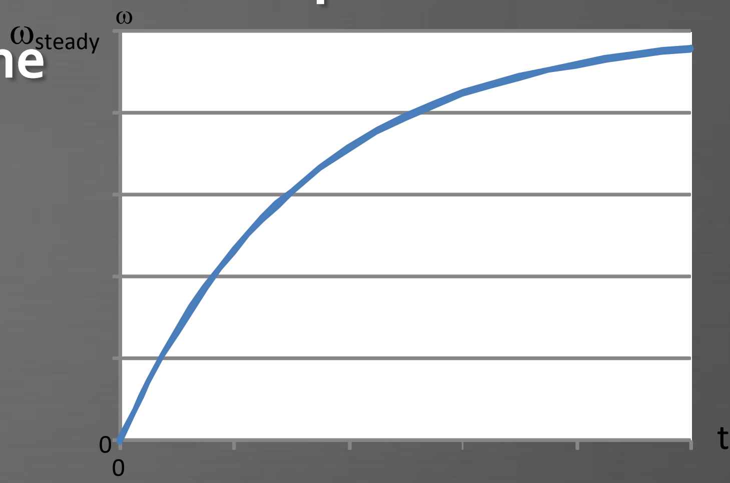
Simplified Equation

- In most DC motors (including ours), the dynamics of the rotor are much slower than those of the RL circuit.
- If L_a is small, we can neglect that term in (1):

$v(t) = R_a i(t) + \cancel{L_a \frac{di(t)}{dt}} + v_m(t)$	(1)	Electrical Model
$v_m(t) = K_e \omega(t)$	(2)	Back EMF
$T(t) = K_t i(t)$	(3)	Torque
$T(t) = J_m \dot{\omega}(t) + b\omega(t)$	(4)	Mechanical Model
$v(t) = \cancel{\frac{L_a}{K_t}\dot{T}(t)} + \frac{R_a}{K_t}T(t) + K_e\omega(t)$	(5)	Substitute (2) and (3) in (1)
$v(t) = \cancel{\frac{L_a J_m}{K_t}\ddot{\omega}(t)} + \left(\frac{\cancel{L_a b} + R_a J_m}{K_t}\right)\dot{\omega}(t) + \left(\frac{R_a b + K_t K_e}{K_t}\right)\omega(t)$	(6)	Substitute (4) in (5)

Simplified Equation

- Now we have a 1st order linear differential equation
  - **Step response:** the response of the system to a step input
  - Step response is an exponential
  - Good approximation to the true behavior



$$v(t) = \left(\frac{R_a J_m}{K_t} \right) \dot{\omega}(t) + \left(\frac{R_a b + K_t K_e}{K_t} \right) \omega(t)$$

System Characterization

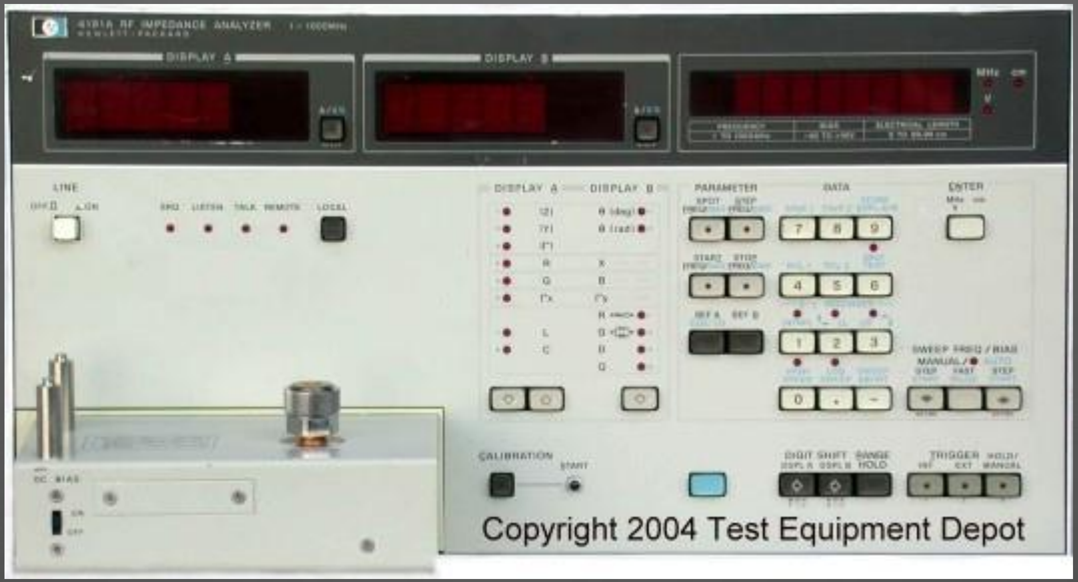
- A system model depends on many physical parameters
  - R_a
  - L_a
  - J_m
  - b
  - K_t
  - K_e
- System characterization is the process of measuring or calculating the values of these parameters

Characterization Setup

- Characterize motor in conjunction with 114.7:1 gear box
  - This accounts for the speed reduction and any friction introduced by the gear box.

Armature Electrical Parameters

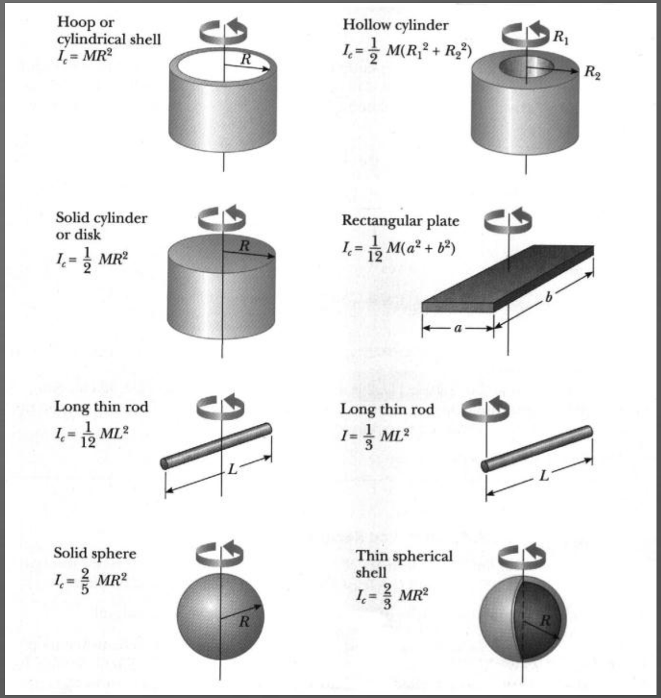
- R_a and L_a can be measured directly with an RLC meter
  - Hook up to the two terminals of the motor.
  - $R_a = 7.56\ \Omega$ $L_a = 2.45$ mH



J_m Calculation

- **Most of the rotational inertia comes from the body & wheels**
  - **Most of the mass of wheels is in the tires**

- **Rotational inertia of a hollow cylinder :**
  - **Inner diameter: R_1**
  - **Outer diameter: R_2**
  - **Mass: m**
  - **$J_w = m(R_1^2 + R_2^2)/2$**



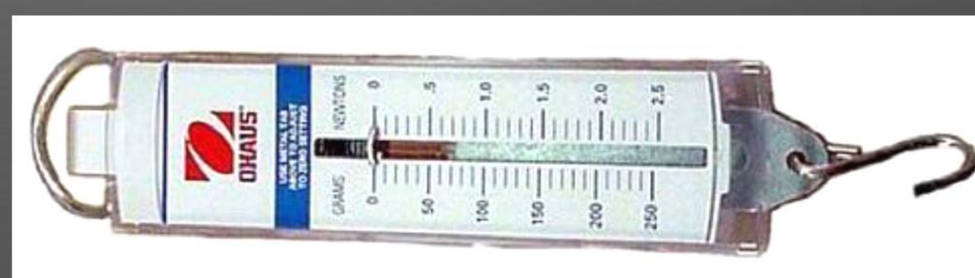
http://members.fortunecity.com/albert66/moment.htm

Total Rotational Inertia

- Measure wheel inner and outer diameter (R_1 and R_2) with ruler
- Measure wheel and body mass (m and M) with a scale
- $J_w = m (R_1^2 + R_2^2)/2$
- $J_b = M R_2^2$
- $J_m = J_w + J_b$
- Complete this calculation on Problem Set 8

Torque Constant

- $K_t = T / i$ when shaft is stationary
- Hold shaft still and measure stall current and stall torque
  - Beware of damaging the axle mounts (torque is substantial)
- Stall current I_{stall}: use ammeter: 0.77 A
- Stall torque:
  - attach a rod of known length d from axle to measuring point
  - attach a spring scale to measuring point and read weight
  - d = 5.5 in
  - scale = 100 g



Torque Constant, Cont.

- $F = mg = (100\ g)\ (9.8\ m/s^2) = 0.98\ N$
- $T_{stall} = Fd = F * (5.5\ in)(0.0254\ m/in) =$ **0.14 N-m**
- $K_t = T_{stall}/i_{stall} = (0.14\ N\text{-}m)\ /\ (0.77\ A) = 0.18\ N\text{-}m/A$

Angular Velocity

- Finding b and K_e require measuring angular velocity
  - Place colored tape on wheel
  - Turn on motor with desired voltage
  - Allow wheel to spin up to full speed
    - Called steady-state operation
  - Measure time for 20 revolutions with a stopwatch
  - Compute revolutions per minute (rpm)
- Angular velocity is in units of radians / sec
    - 1 rpm = (1/60) rev/sec = (2π/60) rad/sec

Friction

- Friction can be computed from the steady-state velocity and current and from the torque constant.

$$J_m \frac{d\omega(t)}{dt} + b\omega(t) = T(t)$$

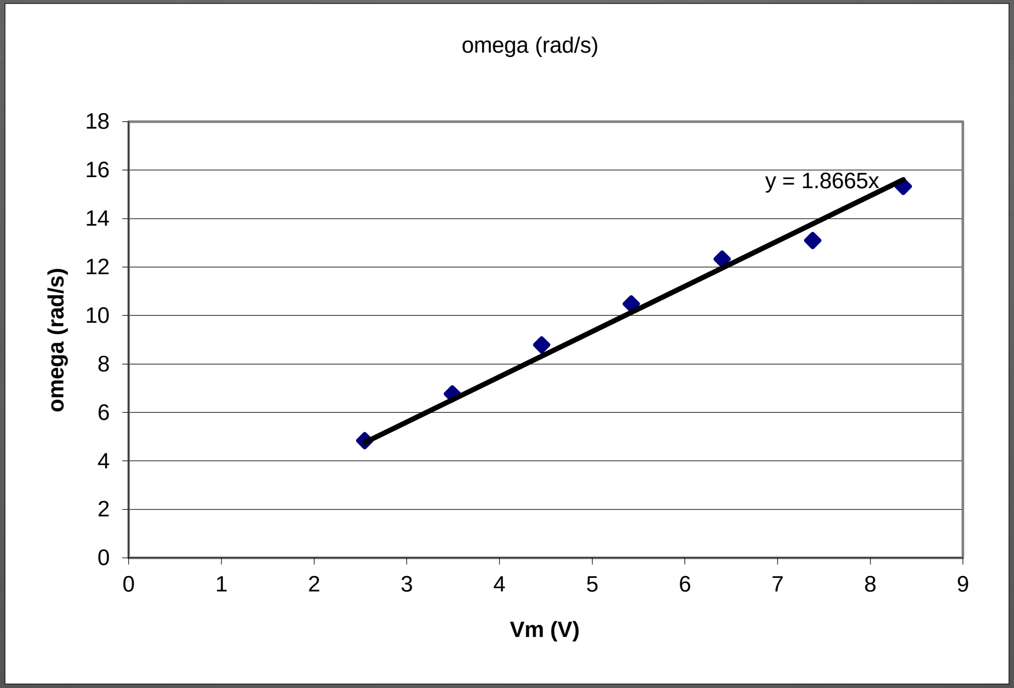
- In steady state, $d\omega/dt = 0$, so equation simplifies.
- Measure ω and i
- Compute T from i and K_t, then find b (PS 8)

Back EMF Constant

- Extract Ke from plot of ω vs. V_m
  - Apply known V
  - Measure steady state ω and I for that V
  - $V_m = V - iR_a$
  - Repeat for various V and generate plot
- Fit a straight line to the plot
  - $K_e = Vm/\omega$ is inverse of slope

Back EMF

- K_e = 1/1.86 = 0.54 V-s / rad



Motor Performance

- Now we have all the parameters for the motor model
- 1^{st} order equation relating speed to voltage
  - When power is switched on, wheels will spin up from 0 to full speed over some amount of time
- See Problem Set 8

Steady-State Speed

- Steady state speed is easier to compute
  - 117.6 RPM @ 7 V battery voltage
- Relate angular velocity to robot's speed
  - $v = \omega R_2$
  - Steady state v:
    - (11.6 rev/min) * (1 min / 60 sec) * (2π rad/rev) * (2.25/2 in)

      = **13.8 in/sec**

Changing Gear Box

- Reducing the gear ratio:
  - Speed:
  - Torque:
  - Controllability:

Increasing Voltage

- Higher V supplied to motor:
  - Speed:
  - Torque:
  - Controllability:

Starting and Stopping

- As compared to a robot moving forward at full power, a robot that alternates full power and halting every 0.5 s will move
  - (a) less than half as fast
  - (b) exactly half as fast
  - (c) more than half as fast