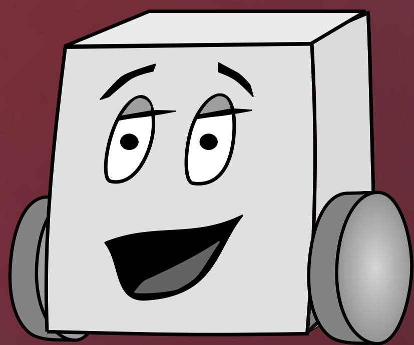

E11 Lecture 14: Capacitors & Inductors

Profs. David Money Harris & Sarah Harris

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Outline

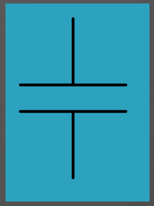
- Frequencies
- Capacitors
- Inductors
- 1st order systems
  - DC Response
  - Step Response

Frequencies

- Consider a signal $x(t) = \cos(\omega t)$
- ω is the frequency of the signal (in units of radians/sec)
- If $\omega = 0$, $x(t) = 1$
  - Zero frequency is a constant signal, called *DC* (direct current)
- If ω is relatively small, signal is called *low frequency*
- If ω is relatively large, signal is called *high frequency*

Capacitor

- A capacitor consists of two conductors separated by an insulator.
- When a voltage V is applied, positive charge +Q accumulates on one plate and negative charge –Q accumulates on the other.
- The capacitance is the ratio of charge to voltage:
  - $Q = CV$
- Units of Farads (farad = coulomb / volt)



Michael Faraday

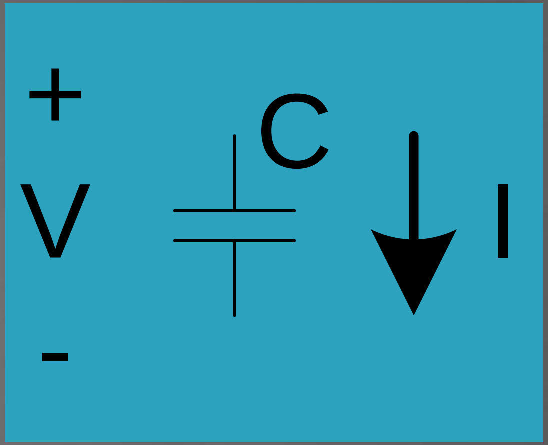
- 1791-1867
- English chemist and physicist
- Poor family, little formal education
  - Didn't know calculus
- One of history's best experimentalists
- Professor at the Royal Institution
- Inventor of motors
- Established the basis for concept of electromagnetic field

en.wikipedia.org/wiki/File:Michael_Faraday_001.jpg

Capacitor I-V Relationship

- Q = CV
- Current is I = dQ/dt
- Hence

$$I = C\frac{dV}{dt}$$



Capacitor Energy Storage

- A capacitor stores energy in the form of the electric field created by the charge.
- If a capacitor is charged from 0 to V_{DD},

  the energy stored is

$$E = \int_0^T P\,dt = \int_0^T VI\,dt = \int_0^T CV\frac{dV}{dt}\,dt = \int_0^{V_{DD}} CV\,dV = \tfrac{1}{2}CV_{DD}^2$$

Capacitor Behavior

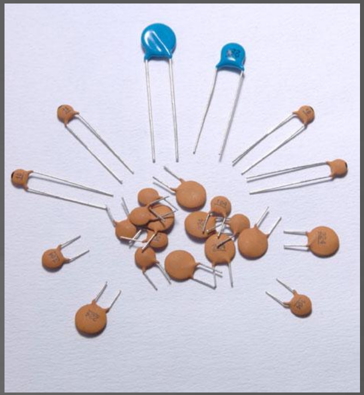
- A capacitor doesn’t like to change its voltage instantly.
  - Requires current
  - Requires energy
- Capacitor looks like:
  - Open circuit at low frequency
  - Short circuit at high frequency

Capacitor Applications

- Store electrical energy
- Stabilize a voltage (such as the power supply)
  - Capacitor opposes changes to its voltage
- Passing only high frequency signals

Capacitor Types

- Ceramic Disk
  - Typical values of 10 pF – 100 nF
  - Cheap and reliable
  - No polarity
- Electrolytic
  - Popular for values > 1 µF
  - Cheap
  - Wide tolerances (~ -50% / +100%)
  - Polarized, can explode if hooked backward
- Tantalum
  - Similar to electrolytic, but smaller and more expensive



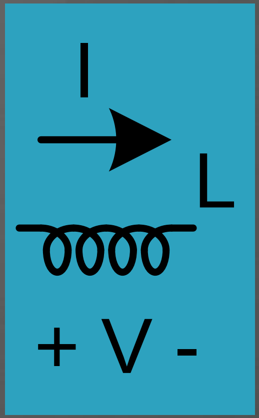
leds-capacitors-manufacturer.com

diytrade.com

Inductor

- An inductor consists of a coil of wire.
- Current flowing in the wire induces a magnetic field
- Changing the magnetic field induces a voltage

$$V = L\frac{dI}{dt}$$



- Inductance L has units of Henries (Volts / (Amperes/sec))

Joseph Henry

- 1797-1878
- American scientist
- Poor family, father died young
- Research was the basis of the telegraph
- First secretary of the Smithsonian

www.photolib.noaa.gov/bigs/pers0124.jpg

Inductor Energy Storage

- An inductor stores energy in the form of the magnetic field created by the current.
- If an inductor has current I flowing,

  the energy stored is

$$E = \int_0^T P\,dt = \int_0^T IV\,dt = \int_0^T IL\frac{dI}{dt}\,dt = \int_0^I LI\,dI = \tfrac{1}{2}LI^2$$

Inductor Behavior

- An inductor doesn't like to change its current instantly.
  - Requires voltage
  - Requires energy
- Inductor looks like:
  - Short circuit at low frequency
  - Open circuit at high frequency

Inductor Applications

- Store magnetic energy
- Magnetically operate electromechanical systems
- Passing only low frequency signals

Inductor Types

- Coils
  - Often wound on iron core to increase magnetic field
  - Typical values of 10 μH – 100 mH
  - Relatively expensive compared to capacitors

http://personal.ee.surrey.ac.uk/Personal/H.M/UGLabs/components/inductors.htm

First Order Systems

- A 1st order system is described by a 1st order differential eq
  - An equation with just a first derivative
- Systems with a single energy storage element are 1st order
  - e.g. a single inductor or capacitor

Example: RC Circuit

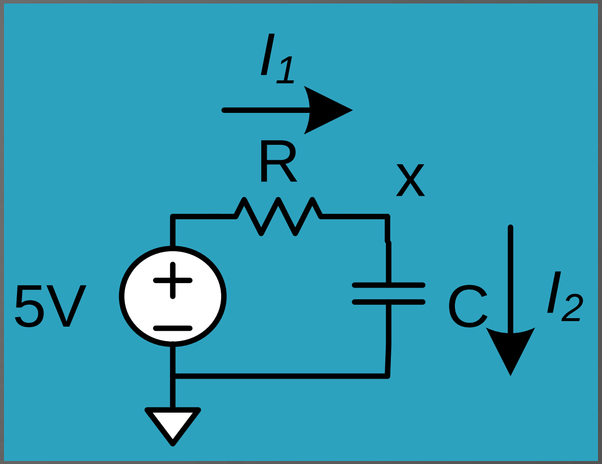
- Apply KCL to find governing equation

$$I_1 = \frac{V - x}{R}$$

$$I_2 = C\frac{dx}{dt}$$

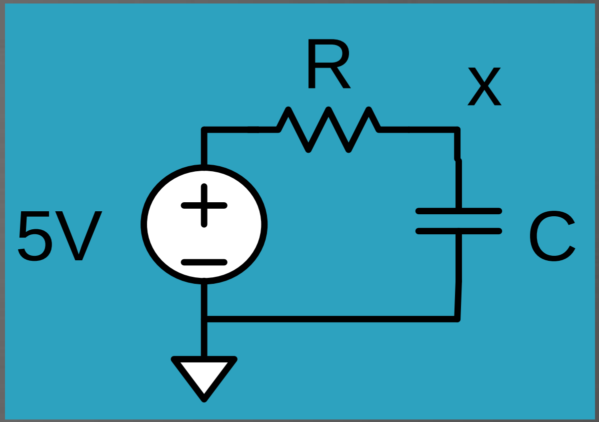
$$I_1 = I_2$$

$$\frac{dx}{dt} + \frac{x}{RC} = \frac{V}{RC}$$



RC Circuit DC Response

- What is the voltage at node x?
- A) 0 V
- B) 2.5 V
- C) 5 V
- D) infinity



Differential Equations

- This is a 1st order differential equation
  - An equation involving a single derivative

$$\frac{dx}{dt} + \frac{x}{RC} = \frac{V}{RC}$$

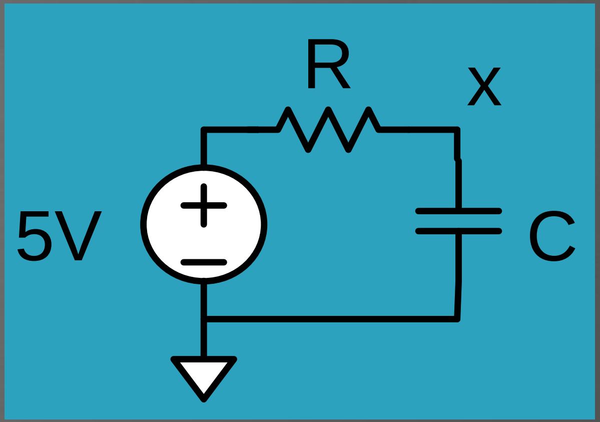
- Solving differential equations
  - Need to know the initial condition (value of x at the start)
  - Guess the form of the answer
    - Use intuition about functions, or past experience
    - All 1st order DiffEqs have solution of the form

$$x(t) = Ae^{\frac{-t}{\tau}} + B$$

  - Substitute the guess into the equation and check
  - Use initial condition to solve for free variable

RC Circuit DC Response

- Two ways to analyze
- Formal: $\frac{dx}{dt} + \frac{x}{RC} = \frac{V}{RC}$
  - V = constant 5 V
    - x must be constant -> dx/dt = 0
    - x = V = 5 V
- Intuitive
  - Capacitor looks like open circuit at DC
  - By voltage divider, $x = V\frac{\infty}{\infty + R} = V$



RC Circuit DC Response

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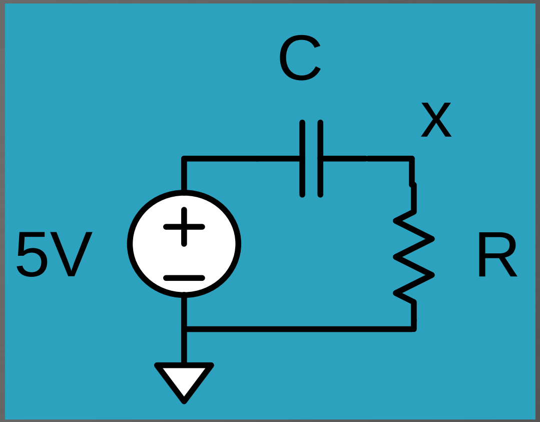
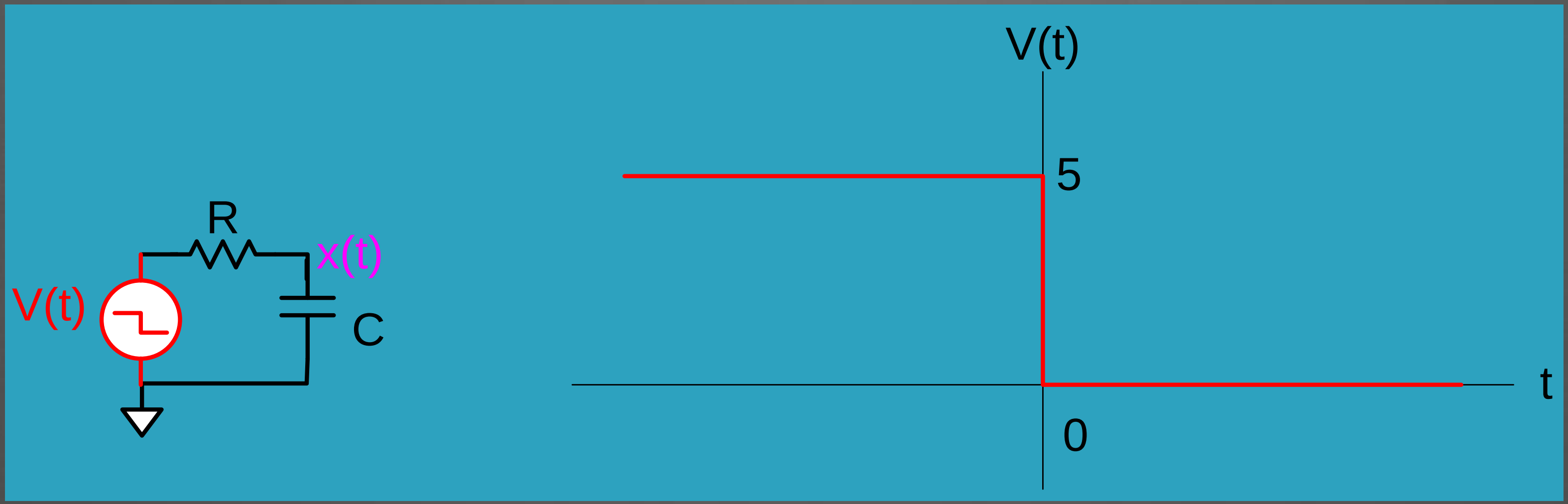


RC Circuit Step Response

- A step is an abrupt change from one value to another.
- What happens if the input voltage steps from 5 to 0 at time t = 0?



RC Circuit Step Response

$$\frac{dx}{dt} + \frac{x}{RC} = \frac{V(t)}{RC}$$

$$V(t) = \begin{cases} 5 & t < 0 \\ 0 & t > 0 \end{cases}$$

- x(t) = 5 for t < 0 (initial condition)
- x(t) for t > 0 must be a function whose derivative is of the same form so it can cancel
- Guess $x(t) = Ae^{\frac{-t}{\tau}} + B$
- Initial condition: x(0) = 5 -> A+B = 5
- Hence $x(t) = 5e^{\frac{-t}{RC}}$

After step

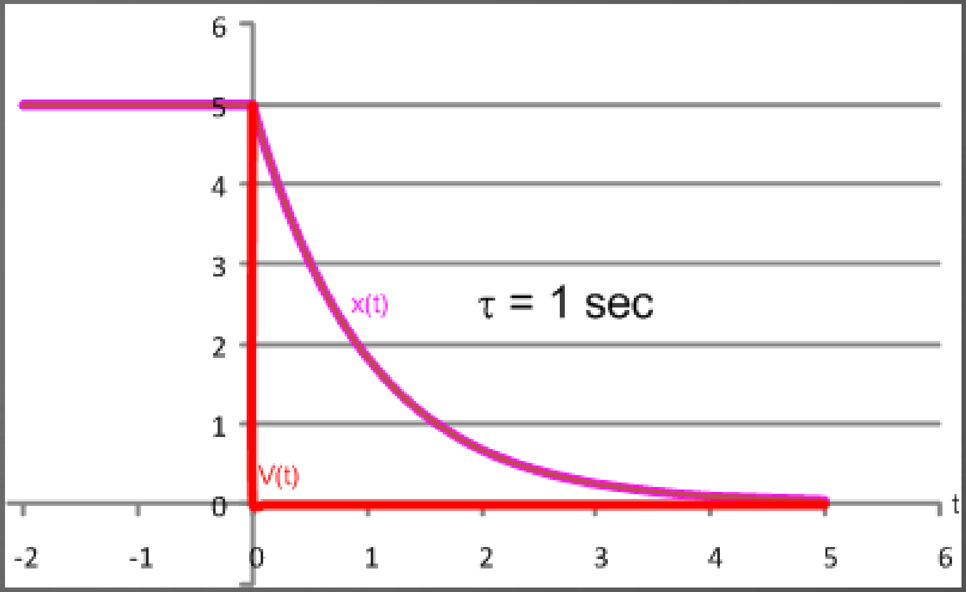
$$\frac{d}{dt}\left(Ae^{\frac{-t}{\tau}} + B \right) + \frac{Ae^{\frac{-t}{\tau}} + B}{RC} = 0$$

$$\frac{-1}{\tau} Ae^{\frac{-t}{\tau}} + \frac{Ae^{\frac{-t}{\tau}} + B}{RC} = 0$$

$$\tau = RC; B = 0$$

RC Circuit Response

- τ is the *time constant*
- Capacitor won't change voltage instantaneously
- τ describes how fast exponential approaches final value
- After 3τ, output is ~ 0

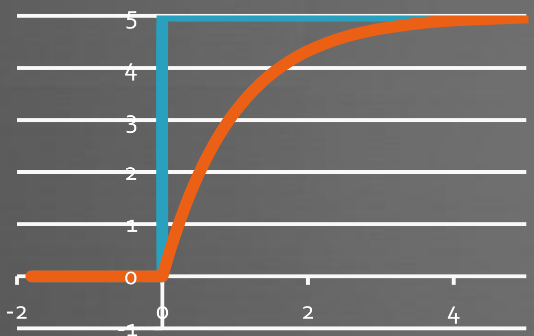


- All 1st order systems have a response in the form of an exponential approaching the final value

Step Response

- What would the RC circuit be to a rising step?

(a)



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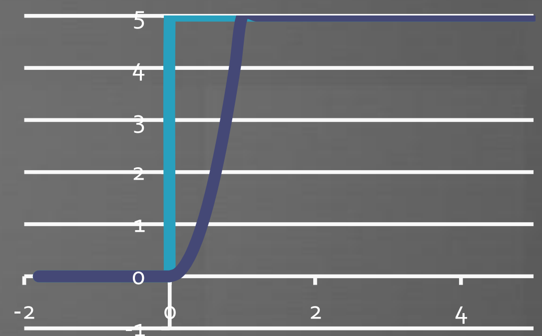
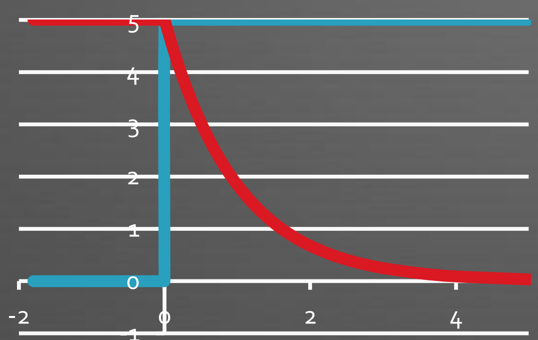


(c)



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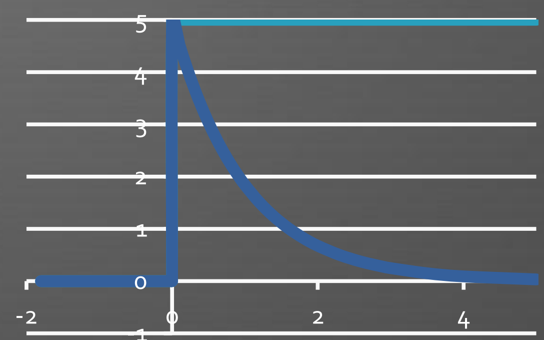

RC Circuit Step Response

$$\frac{dx}{dt} + \frac{x}{RC} = \frac{V(t)}{RC}$$

$$V(t) = \begin{cases} 0 & t < 0 \\ 5 & t > 0 \end{cases}$$

- Solution is of the form $x(t) = Ae^{\frac{-t}{\tau}} + B$
- Initial condition: x(0) = 0 -> A+B = 0
- After step:

$$\frac{d}{dt}\left(Ae^{\frac{-t}{\tau}} + B\right) + \frac{Ae^{\frac{-t}{\tau}} + B}{RC} = 5$$

$$\frac{-1}{\tau} Ae^{\frac{-t}{\tau}} + \frac{Ae^{\frac{-t}{\tau}} + B}{RC} = 5$$

$$\tau = RC; B = 5$$

- Hence,

$$x(t) = 5 - 5e^{\frac{-t}{RC}} = 5\left(1 - e^{\frac{-t}{RC}}\right)$$