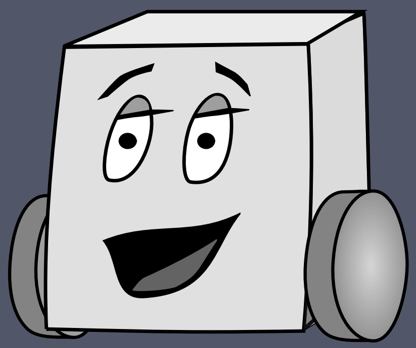
E11 Lecture 13: Motors



Professor Lape
Fall 2010

Overview

- How do electric motors work?
  - Electric motor types and general principles of operation
- How well does your motor perform?
  - Torque and power output
  - Motor modeling
  - Gear ratios

E11 Announcements

- Solutions to programming assignments posted on E11 programming website (linked from regular E11 site)

- Problem Set 5 (Energy) will be due on Wednesday, 27 October
  - Tutoring will be available in the LAC 1-3 PM on Saturday, 23 October
  - Can also come to office hours (Prof Lape has 9-11 AM Tues and Wed)

Reminder: E11 Grading

E11 is pass/fail. To pass the class, you are expected to:

- regularly attend class and lab;
- complete all but one of the weekly labs;
- complete at least six of the seven homework assignments;
- deploy an operational autonomous vehicle to play Capture the Flag;
- make a presentation about your vehicle; and
- complete a final report documenting your vehicle.

(Some) Electric Motor Types

General Motor types:

- **DC Motor**
- **AC Motor**
- **Universal Motor**:
- Can operate on either DC or AC

Controlled Motors:

- **Servo Motors**:
  - Use feedback to control position of motor
  - Can rotate continuously
- **Stepper Motors**:
  - "Step" from one position to the next
  - Do not require feedback to run

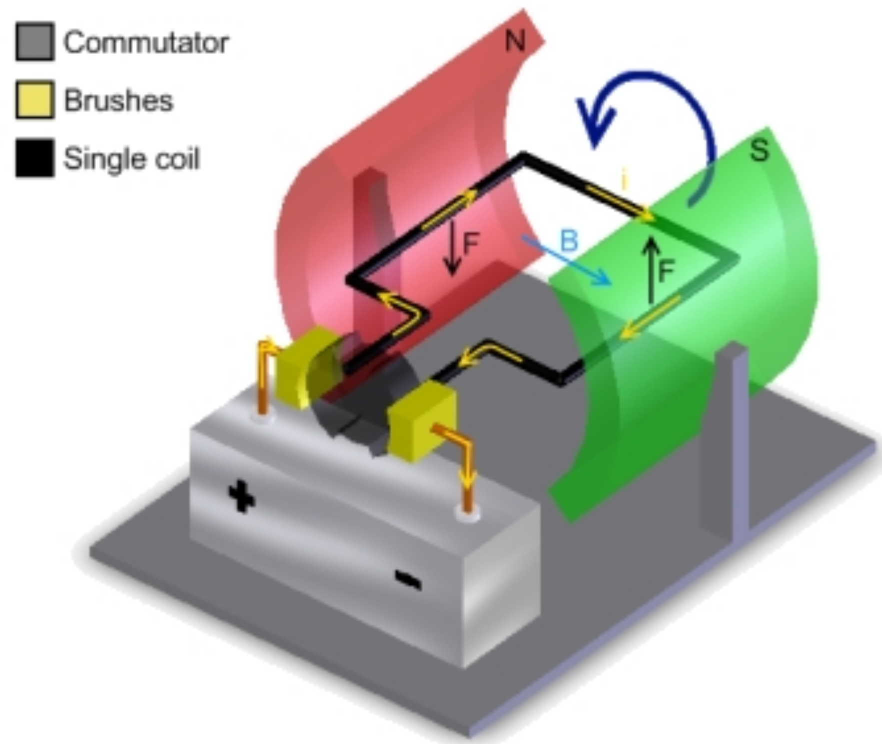
How does a DC Motor work?

1. The **stator** generates a stationary magnetic field surrounding the rotor.

2. The **rotor/armature** is composed of a coil which generates a magnetic field when electricity flows through it.

3. The **brushes** provide mechanical contact between the rotor and the commutators and help switch polarity of rotor windings.

4. **Commutators** reverse the current every half a cycle to keep the motors turning.

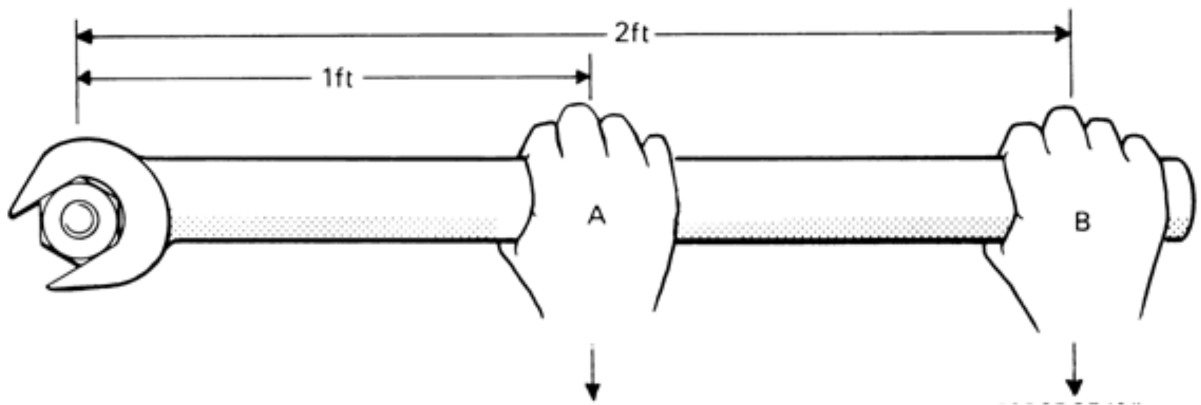


http://humanoids.dem.ist.utl.pt/servo/overview.html

i-Clicker #1

- How does the force required to tighten the bolt compare at point A and point B?

A. $F_A > F_B$
B. $F_B > F_A$
C. $F_B = F_A$



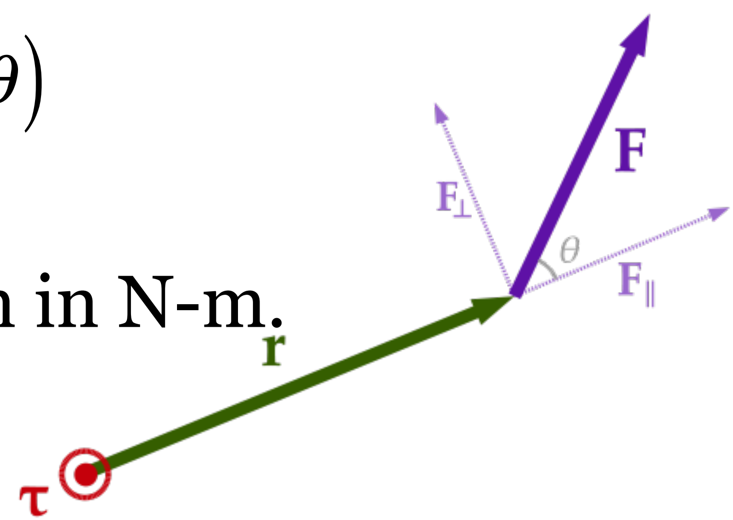
http://mdmetric.com/tech/torqcht2.htm

Torque

- **Torque**, or moment of the force, can be loosely thought of as the turning or twisting action of a force *F*.

$$T = r \cdot F_{\perp} = r\left(F\sin\theta\right)$$

- In SI units, torque is given in N-m.



http://en.wikipedia.org/wiki/File:Torque,_position,_and_force.svg

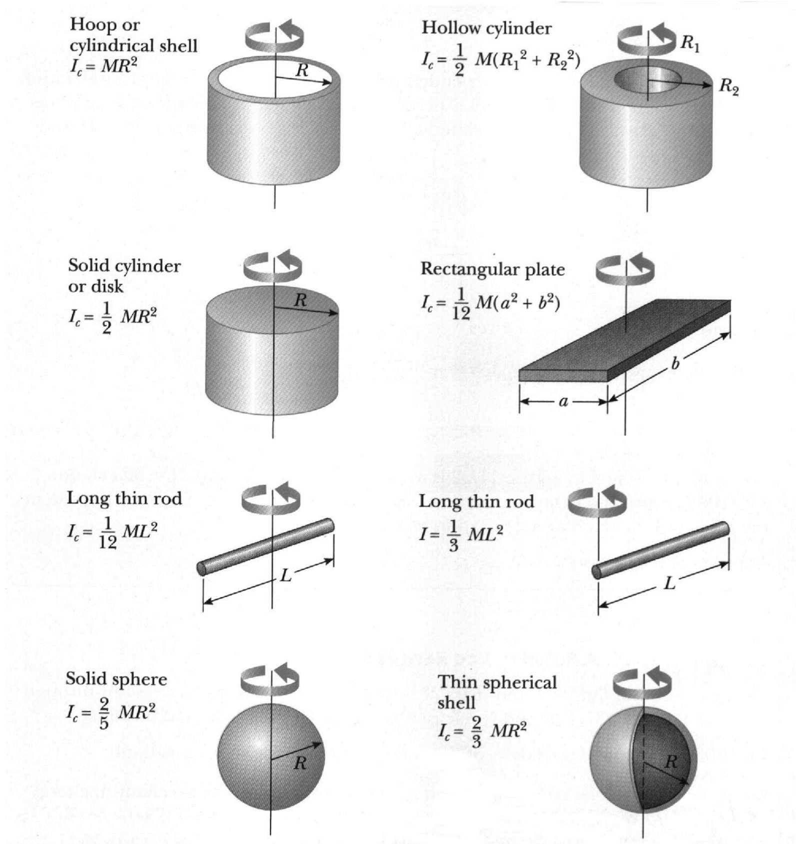
Newton's Second Law for Rotation

- $\Sigma F = ma$ becomes

$$\sum T = J_m \alpha = J_m \frac{d\omega}{dt}$$

where:

- J_m is the **moment of inertia** (how mass is distributed about axis of rotation)
- α is the **angular acceleration**
- ω is **angular velocity**



http://members.fortunecity.com/albert66/moment.htm

Power for rotational motion

- Recall $W = \int F\, dx$

- The rotational equivalent is $W = \int T\, d\theta$

- To find power, $P = \dfrac{dW}{dt} = T\dfrac{d\theta}{dt} = T\omega$

Motor Torque: Mechanical Model

- If the friction in proportional to the angular velocity, we can use Newton's 2nd Law for Rotation to write the governing equation for the motor:

$$T(t) = J_m \frac{d\omega(t)}{dt} + b\omega(t)$$

where *b* is the friction coefficient.

i-Clicker #2

- If there were no friction present in the motor and gear train, robot wheels attached to the motor would:

A. Decelerate continuously
B. Turn at a constant angular velocity
C. Accelerate continuously

i-Clicker #3

- If the wheels driven by a real motor-gear train system are moving at a constant angular velocity,

A. No torque is required.
B. Motor torque must exactly balance out friction.
C. Motor torque must exceed friction.

i-Clicker #4

- If the coefficient of friction for a motor is 0.5 N-m/rpm, and it exerts 100 N-m of torque at steady state, what is its steady angular velocity?

A. 50 rpm
B. 100 rpm
C. 200 rpm
D. Not enough information to determine

RL Circuit Model of DC Motor

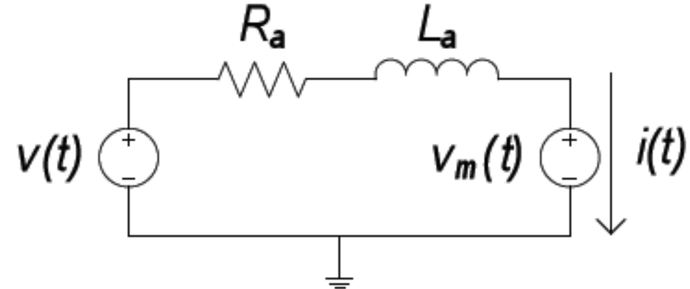
The electrical model of the DC motor is generated considering the following:

- The armature coil has a resistance R_a and an inductance L_a.
- The spinning rotor induces an additional voltage in the coil called the *back emf* (electromotive force), $v_m(t)$ that is proportional to the angular velocity.

$$v_m(t) = K_e \omega(t)$$

- The torque applied to the rotor is proportional to the current flowing through the coil.

$$T(t) = K_t i(t)$$



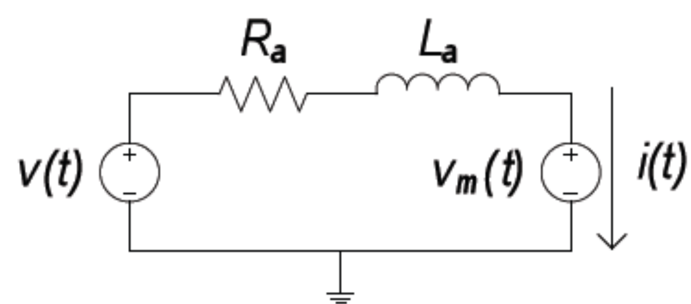
RL Circuit Model of DC Motor, cont.

- We can show that for this circuit,

$$v(t) = R_a i(t) + L_a \frac{di(t)}{dt} + v_m(t)$$

- Given $v_m(t) = K_e \omega(t)$ and $T(t) = K_t i(t)$, this can be rewritten as

$$v(t) = \frac{R_a}{K_t} T(t) + \frac{L_a}{K_t} \frac{dT}{dt} + K_e \omega(t)$$



- Now we have a relationship between applied voltage, torque, and angular velocity!

How fast can your motor go?

- In most DC motor applications, the dynamics of the rotor are much slower than those of the RL circuit, making the inductance L_a negligible. This makes the RL circuit model simply

$$v(t) = \frac{R_a}{K_t} T(t) + K_e \omega(t)$$

- Combining with the mechanical model $T(t) = J_m \frac{d\omega(t)}{dt} + b\omega(t)$ gives

$$v(t) = \left(\frac{R_a J_m}{K_t} \right) \frac{d\omega(t)}{dt} + \left(\frac{R_a b + K_t K_e}{K_t} \right) \omega(t)$$

Can solve to find angular velocity as a function of time given values for all constants and applied voltage *v*.

Finding ω(t) for your motor

- The combined electrical and mechanical governing equation is

$$v(t) = \left(\frac{R_a J_m}{K_t}\right)\frac{d\omega(t)}{dt} + \left(\frac{R_a b + K_t K_e}{K_t}\right)\omega(t)$$

can be rewritten for the case of constant applied voltage υ as

$$\frac{d\omega(t)}{dt} + \frac{\omega(t)}{\tau} = C$$

where the constants τ and C can be determined from constants in the governing equation.

$$\tau = \frac{R_a J_m}{R_a b + K_t K_e} \qquad C = \frac{v K_t}{R_a J_m}$$

Finding ω(t) for your motor II

- To solve this DE, we can guess that it can be solved with an exponential, or use the method of separation of variables.
- If the motor is not turning initially ($\omega_o = 0$) and a voltage is applied at t = 0, the angular velocity will respond as:

$$\int_0^{\omega(t)} \frac{d\omega}{C - \frac{1}{\tau}\omega} = \int_0^t dt$$

$$-\tau\left[\ln\left(C - \frac{1}{\tau}\omega(t)\right) - \ln(C - 0)\right] = t$$

$$\ln\left[\frac{C - \frac{1}{\tau}\omega(t)}{C}\right] = -\frac{t}{\tau}$$

$$C - \frac{1}{\tau}\omega(t) = Ce^{-t/\tau}$$

$$\omega(t) = C\tau\left[1 - e^{-t/\tau}\right]$$

Friendly Reminder:

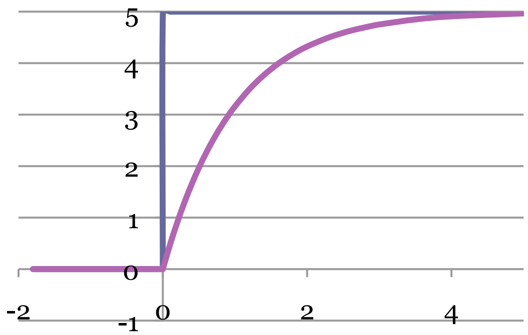
$$\tau = \frac{R_a J_m}{R_a b + K_t K_e}$$

$$C = \frac{vK_t}{R_a J_m}$$

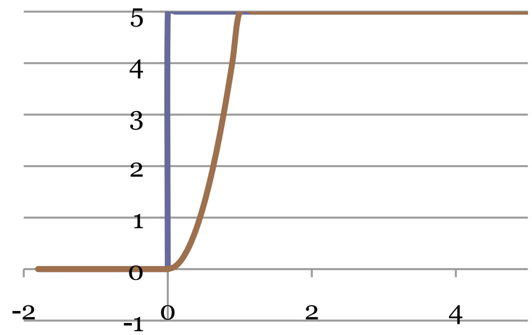
Motor Step Response

- How would your angular velocity change in response to a step increase in applied voltage?

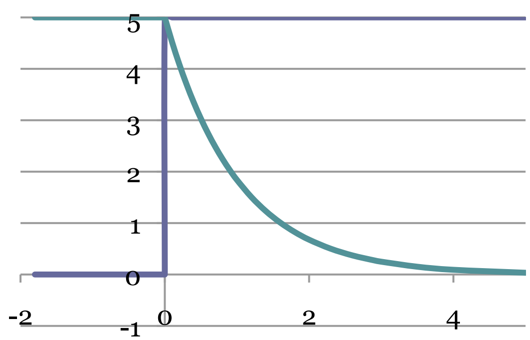
(a)



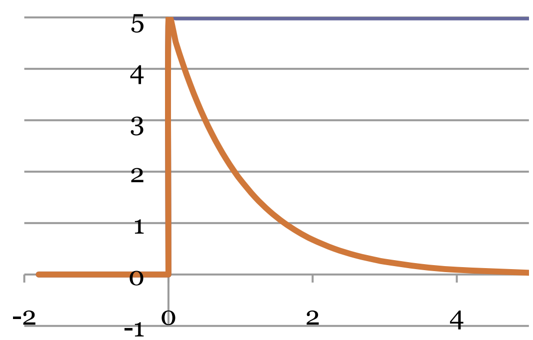
(b)



(c)



(d)



i-Clicker #5

- If the time constant τ is large, when a step change in voltage v is applied to the system (e.g. when you plug your battery into your motor) the angular velocity will reach its maximum:

A. Quickly.
B. Slowly.

Finding ω(t) for your motor II

- For your motors, the product of electrical and mechanical resistances is much less than the torque and back emf constants, so

$$\tau = \frac{R_a J_m}{R_a b + K_t K_e} \approx \frac{R_a J_m}{K_t K_e}$$

Ratio of resistance times inertia to product of torque and back emf constants

$$C\tau \approx \frac{vK_t}{R_a J_m} \cdot \frac{R_a J_m}{K_t K_e} = \frac{v}{K_e}$$

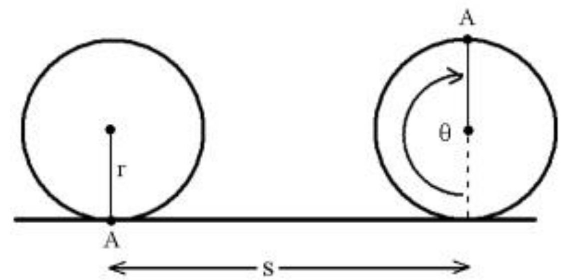
Ratio of applied voltage and back emf constant, where the back emf constant $=v_m/\omega$

$$\omega(t) = C\tau\left[1 - e^{-t/\tau}\right] = \frac{v}{K_e}\left[1 - e^{-t/\tau}\right]$$

How does angular velocity relate to linear velocity?

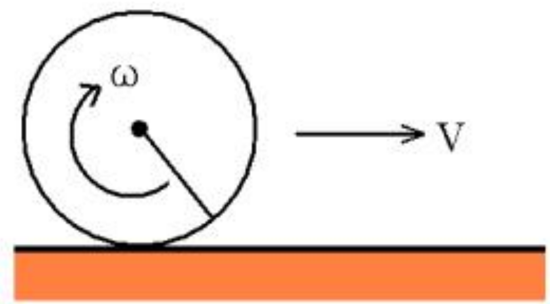
- If the motors (and therefore the wheel) is spinning at angular velocity ω, the tangential distance *s* is

$$s = r\theta$$



- Plugging into the definition of ω,

$$\omega \equiv \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt}$$



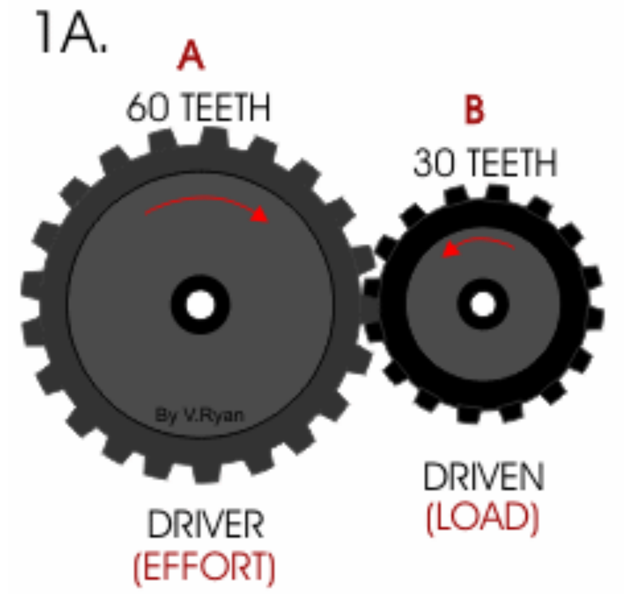
- so the translational velocity

$$V = \frac{ds}{dt} = \omega r$$

http://www.phy.cmich.edu/people/andy/Physics110/Book/Chapters/Chapter6_files/image040.jpg

Gear Trains and Ratios

- **Gear trains** reduce speed and magnify torque.
- The **gear ratio** is the ratio of number of teeth on driver gear A to those on driven gear B:



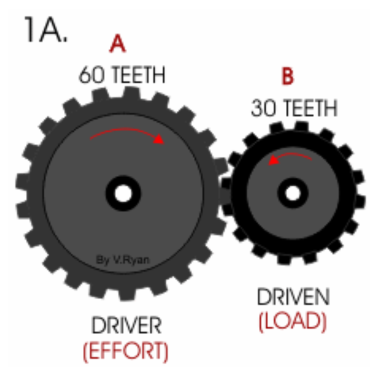
$$GR = \frac{\text{number of teeth on gear A}}{\text{number of teeth on gear B}}$$

Gear Ratio and Angular Velocity

- The gear ratio is also proportional to the ratio of radii:

$$GR = \frac{r_A}{r_B}$$

- The surface speeds at the point of contact of the gears must be identical, so



$$v_A = v_B \Rightarrow \omega_A r_A = \omega_B r_B$$

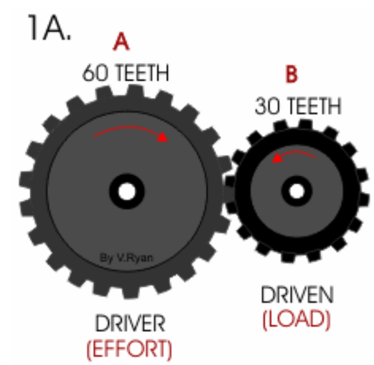
- Therefore,

$$GR = \frac{n_A}{n_B} = \frac{r_A}{r_B} = \frac{\omega_B}{\omega_A}$$

Gear Ratio and Torque

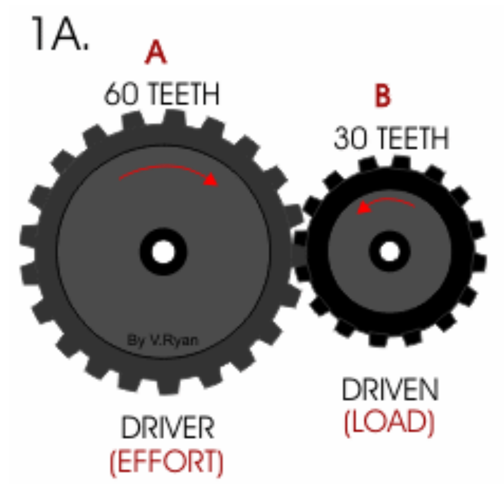
- If we neglect losses to friction, the power is transmitted across the gear train unchanged.

$$P_A = P_B$$
$$T_A \omega_A = T_B \omega_B$$



i-Clicker #6

- If gear A spins at 100 rps with a 5 W power output and frictional losses can be neglected, what is the torque exerted by gear B?



A. 0.025 N-m
B. 0.05 N-m
C. 0.1 N-m
D. Not enough information to determine

i-Clicker #6 Solution

- First, we can find the angular velocity of gear B using the gear ratio:

$$GR = \frac{n_A}{n_B} = \frac{60}{30} = \frac{\omega_B}{\omega_A} = \frac{\omega_B}{100\text{ rps}} \Rightarrow \omega_B = 200\text{ rps}$$

- Then, since the power is transmitted across the gears,

$$P_A = T_A\omega_A = T_B\omega_B = 5\text{ W}$$

$$\Rightarrow T_B = \frac{5\text{ W}}{200\text{ rps}} = 0.025\text{ N-m}$$