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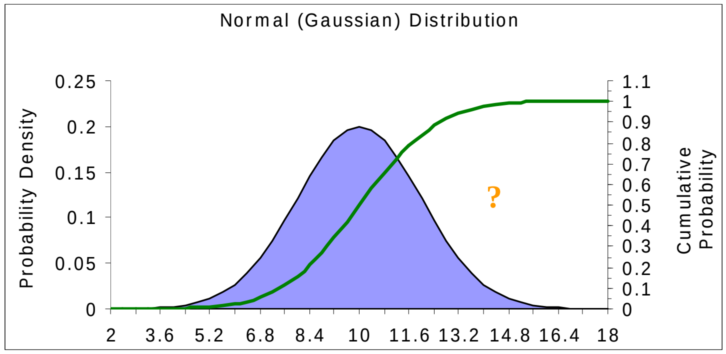

The Black-Scholes Model

... pricing options and calculating Greeks

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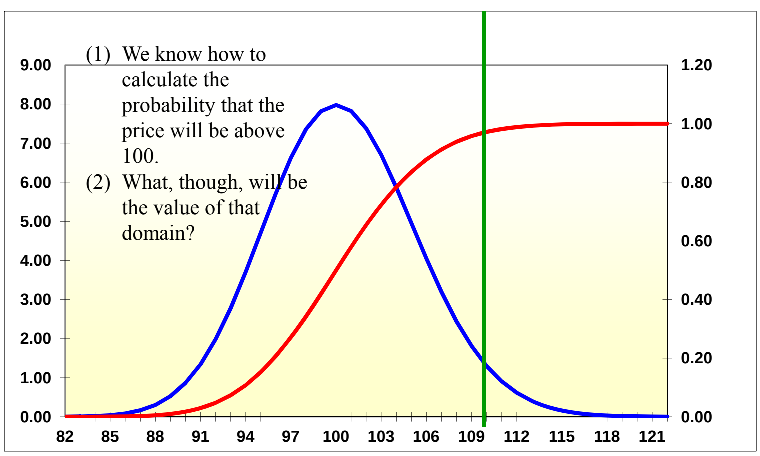
Conceptually calculating what a 110 OTM call option should be worth if the present price of the stock is 100 ...



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How Black-Scholes works ...

The Black-Scholes model is used to price European options (which assumes that they must be held to expiration) and related custom derivatives. It takes into account that you have the option of investing in an asset earning the risk-free interest rate.

It acknowledges that the option price is purely a function of the volatility of the stock's price (the higher the volatility the higher the premium on the option).

Black-Scholes treats a call option as a forward contract to deliver stock at a contractual price, which is, of course, the strike price.

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The Essence of the Black-Scholes Approach

- Only volatility matters, the *mu* (drift) is not important.
- The option's premium will suffer from time decay as we approach expiration (Theta in the European model).
- The stock's underlying volatility contributes to the option's premium (Vega).
- The sensitivity of the option to a change in the stock's value (Delta) and the rate of that sensitivity (Gamma) is important [these variables are represented mathematically in the Black-Scholes DE, next lecture].
- Option values arise from arbitrage opportunities in a world where you have a risk-free choice.

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The Black-Scholes Model: European Options

$$C = SN(d_1) - Ke^{-r\left(t/365\right)}N(d_2)$$

C = theoretical call value
S = current stock price
N = cumulative standard normal probability dist.
t = days until expiration
K = option strike price
r = risk free interest rate
σ = daily stock volatility

$$d_1 = \frac{\ln(S/K) + \left(r/365 + \sigma^2/2\right)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(S/K) + \left(r/365 - \sigma^2/2\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Note: Hull's version (13.20) uses annual volatility. Note the difference.

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Breaking this down ...

$$C = SN(d_1) - Ke^{-r\left(t/365\right)}N(d_2)$$

This term discounts the price of the stock at which you will have the right to buy it (the strike price) back to its present value using the risk-free interest rate. Let's assume in the next slide that r = 0.

$$d_1 = \frac{\ln(S/K) + \left(r/365 + \sigma^2/2\right)t}{\sigma\sqrt{t}}$$

Dividing by this term (the standard deviation of stock's daily volatility adjusted for time) turns the distribution into a standard normal distribution with a standard deviation of 1.

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... or simplifying it some

$$CP = SP \times \Pi(d_1) - STR \times \Pi(d_2)$$

This is the absolute log growth difference between the strike price and the stock price.

... assume that r is 0 and t is 1:

$$d_1 = \frac{\ln\left(SP/STR\right) + \sigma^2/2}{\sigma}$$

We are calculating the cumulative probability to this standard normal point.

This normalizes it to standard normal (the numerator is now "number of standard deviations."

μ is zero so this is the log-normal zero mean adjustment

$C = S \times N(d_1) - K \times N(d_2)$ (assuming ***r*** to be 0)

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... and some more

This term, our ***x*** of two slides ago, represents the **spread in continuous growth terms** between the stock price and the strike price, and when normalized by the denominator, the spread as the number of standard deviations. For example, if ***S*** = 110 and ***K*** = 100 and volatility = 10%, then this terms equals 9.5%, or about one standard deviation. ***x*** > 0 for **itm calls** and **otm puts** and ***x*** < 0 for **otm calls** and **itm puts**.

$$d_1 = \frac{\ln(S/K) + (r/365 + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(S/K) + (r/365 - \sigma^2/2)t}{\sigma\sqrt{t}}$$

This term has the effect of removing the bias.

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Using the Black-Scholes Model

There are variations of the Black-Scholes model that prices for dividend payments (within the option period). See Hull section 13.12 to see how that is done (easy to understand). However, because of what is said below, you really can't use Black-Scholes to estimate values of options for dividend-paying American stocks

There is no easy estimator for American options prices, but as Hull points out in chapter 9 section 9.5, with the exception of exercising a call option just prior to an ex-dividend date, "it is never optimal to exercise an American call option on a non-dividend paying stock before the expiration date."

The Black-Scholes model can be used to estimate "***implied volatility***". To do this, however, given an actual option value, you have to iterate to find the volatility solution (see Hull's discussion of this in 13.12). This procedure is easy to program and not very time-consuming in even an Excel version of the model.

For those of you interest in another elegant implied volatility model, see Hull's discussion of the IVF model in 26.3. There you will see a role played by delta and vega, but again you would have to iterate to get the value of the sensitivity of the call to the strike price.

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Calculating implied volatility with B/S:

$$d_1 = \frac{\ln\left(SP/STR\right) + \sigma^2/2}{\sigma}$$

Very easy to do:
Once Black-Scholes is structured, you can use an iterative technique to solve for σ.

Name: Gary R. Evans
Date: October 27, 2011

Put Option
Implicit Daily Volatility (IDV) Calculator

Stock	Symbol:	DIA
	Price:	121.60
Put	Month:	Dec
	Strike:	120.00
	Price:	3.250
	Expiration date:	12/17/2011
	Interest rate:	0.0100
	Days to maturity today:	51
	Days to maturity override:	51
	Implied daily volatility:	0.01452
	One-day time decay:	0.045

Version 3.4 Aug 16, 2011

Calculate

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VBasic iterative technique used in IDV master

```
'Below is the actual calculation of implied volatility.
'The Ringer is for testing temporary values in construction only.
  '
    Do
      CIPD = CIPD + 0.00001
      DeNom = Log(StockPR / StrikePR) + ((IntRR / 365) + (CIPD ^ 2) / 2) * DTMR
      DurVol = CIPD * DTMR ^ 0.5
      DND1 = WorksheetFunction.NormSDist(DeNom / DurVol)
      DND2 = WorksheetFunction.NormSDist(DeNom / DurVol - DurVol)
      Ringer = Exp(-IntRR * DTMR / 365)
      TempCallPR = StockPR * DND1 - StrikePR * Exp(-IntRR * DTMR / 365) * DND2

    Loop Until TempCallPR >= CallPR

  'Command below writes a value back to a named designated cell
  Range("CIPD").Value = CIPD
```

Calculating IDV for strangles (V. 3.3)

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Name: Gary R. Evans
Date: March 30, 2012

Strangle Implied Daily Volatility Calculator

Stock Symbol:	DIA	Interest rate:	
Stock Price:	131.680		0.010
	CALL	**PUT**	
Month:	Apr	Apr	
Strike:	134.00	130.00	
Expiration:	4/21/12	4/21/12	
Price:	0.460	0.980	
Days to maturity:	22	22	
DTM override:	22	22	
Implied daily volatility:	0.00516	0.00702	
One-day time decay:	0.022	0.035	

Version 3.3 August 16, 2011

Calculate

One Year:	
Average DGR:	0.00038
Standard Deviation:	0.01299
Average ABS DCGR:	0.00909
60 day:	
Average DGR:	0.00109
Standard Deviation:	0.00565
Average ABS DCGR:	0.00415

Gary R. Evans @PITraders 30 Mar
Weekend strangle DIA 131.68 Call Apr 21 134 @ 0.46 CIDV 0.0052 Put 130 @ 0.98 PIDV 0.007, DIA HIDV 60day 0.0055, time decay 6 cents daily.

Gary R. Evans @PITraders 1h
VIX not cooperative on weekend strangle: Sold DIA Apr 134 call 0.55, paid 0.46. 130 put 0.74, paid 0.99, loss 0.14 gross, mostly time decay.

Example: March 30, 2012 weekend strangle

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=LN(SP/KP)+(IR+(DV*DV)/2)*(DTM/365)

=LN(SP/KP)+((IR/365)+(DV*DV)/2)*DTM

An example ...

Consider an itm option with 20 days to expiration. The strike price is 105 and the price of the stock is 100 and the stock has an daily volatility of 0.02. Assume an interest rate of 0.01 (1% annual).

$$d_1 = \frac{\ln(100/105) + \left(r/365 + 0.02^2/2\right)20}{0.02\sqrt{20}} = -0.49464$$

$$d_2 = d_1 - 0.02\sqrt{20} = -0.58409$$

$$C = 100N(-0.04424) - 105e^{-0.01\left(20/365\right)}N(-0.58409) = 1.70$$

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Using an Option Value Calculator to Calculate this same Value

Call Option Price Calculator (Daily Volatility)

Stock symbol:	Trial
Call option:	May
Date Today:	4/26/2011
Expiration Date:	5/16/2011
DTM:	20
Stock Price:	100.00
Strike Price:	105.00
Daily Volatility:	0.0200
Interest Rate:	0.010
Time:	20
d1 Numerator:	-0.04424
Duration Volatility:	0.08944
Delta N(d1):	0.3104
N(d2):	0.2796
Option Price:	1.70
Option Premium:	1.70

NUM

=LN(SP/KP)+((IR/365)+(DV*DV)/2)*DTM

DUV

=NORMSDIST(NUM/DUV)