Correlation ...  
... possibly the most important and least understood topic in finance
The first exam ...  

<table>
<thead>
<tr>
<th>Score Range</th>
<th>Grade</th>
<th>Count</th>
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<tbody>
<tr>
<td>184+</td>
<td>A</td>
<td>24</td>
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<td>180 - 184</td>
<td>A-</td>
<td>5</td>
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<tr>
<td>175 - 179</td>
<td>B+</td>
<td>4</td>
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<td>165 - 174</td>
<td>B</td>
<td>4</td>
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<td>160 - 164</td>
<td>B-</td>
<td>1</td>
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<tr>
<td>155 - 159</td>
<td>C+</td>
<td>1</td>
</tr>
<tr>
<td>145 - 154</td>
<td>C</td>
<td>1</td>
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<tr>
<td>140 - 144</td>
<td>C-</td>
<td>1</td>
</tr>
<tr>
<td>Below</td>
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</tbody>
</table>

What can I say ... you are a bunch of geniuses.
... your spread should look like this above, with the results shown on the next page.
The results from HW2 ...

<table>
<thead>
<tr>
<th></th>
<th>One Year:</th>
<th>60 day:</th>
<th>30 Day:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCGR</td>
<td>0.00073</td>
<td>0.00022</td>
<td>0.00009</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.00713</td>
<td>0.00679</td>
<td>0.00773</td>
</tr>
<tr>
<td>Min DCGR</td>
<td>-0.02511</td>
<td>-0.02157</td>
<td>-0.02157</td>
</tr>
<tr>
<td>Max DCGR</td>
<td>0.02132</td>
<td>0.01691</td>
<td>0.01691</td>
</tr>
<tr>
<td>Min Norm DCGR</td>
<td>-3.62239</td>
<td>-3.20994</td>
<td>-2.80085</td>
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<tr>
<td>Max Norm DCGR</td>
<td>2.88677</td>
<td>2.45967</td>
<td>2.17469</td>
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</tbody>
</table>

One year data
- Correlation: 0.39446
- Ratio of SD: 2.07080
- Beta: 0.87090

30 day data
- Correlation: 0.65438
- Ratio of SD: 1.17878
- Beta: 0.77137

Well, now, this is very interesting, isn’t it? Is there anything useful here?

SD with 3 5-sigma observations removed: 0.010789

So what are these 7-sigma??

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</thead>
<tbody>
<tr>
<td>Mean DCGR</td>
<td>0.00032</td>
<td>-0.00038</td>
<td>0.00180</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.01575</td>
<td>0.01828</td>
<td>0.00912</td>
</tr>
<tr>
<td>Min DCGR</td>
<td>-0.11605</td>
<td>-0.11605</td>
<td>-0.01609</td>
</tr>
<tr>
<td>Max DCGR</td>
<td>0.11891</td>
<td>0.02163</td>
<td>0.02077</td>
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<tr>
<td>Min Norm DCGR</td>
<td>-7.38955</td>
<td>-6.32713</td>
<td>-1.96189</td>
</tr>
<tr>
<td>Max Norm DCGR</td>
<td>7.53017</td>
<td>1.20364</td>
<td>2.08090</td>
</tr>
</tbody>
</table>

CSCO
What does this tell you about strangle policy?

Q: How would you write a scripting screener to find strangle candidates?

Original PIT debate (Asaf Bernstein and Jason Christianson), does the adjusted SD mean anything?
The *Beta* interpretations ...

<table>
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<tr>
<th>One year data</th>
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<tbody>
<tr>
<td>Correlation:</td>
<td>0.39446</td>
</tr>
<tr>
<td>Ratio of SD:</td>
<td>2.20780</td>
</tr>
<tr>
<td>Beta:</td>
<td>0.87090</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>30 day data</th>
<th></th>
</tr>
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<td>Correlation:</td>
<td>0.65438</td>
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</tr>
<tr>
<td>Beta:</td>
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1. We can see why we need to separate the Beta into the two components and leave them that way.

2. The ratio of SD is really indicative of the relative volatilities, BUT

3. The traditional Beta matters because if you do add an uncorrelated asset to a portfolio, the variance of the portfolio is reduced, which means the portfolio has less risk!! That is a big issue!
Some elementary starting points ...

We are talking about a series of random variables and weighted random variables that fit a Gaussian distribution as we have defined it. Notationally ...

\[ X, \text{ etc } \sim N(\mu_x, V_x) \]

and

\[ V_x = \sigma^2 \]

where

\[ V_x = \frac{\sum_{i=1}^{n} (X_i - \mu_x)^2}{n} \]

it turns out that if \( Y = e^X \) then \( Y \) has a lognormal distribution.
2-asset Portfolio Variance Sums

Variance is purely additive if two variables are strictly independent:

\[ V(x + y) = V(x) + V(y) + 2COV(x, y) \]

remembering that Covariance is equal to the Correlation Coefficient (0 if no perfectly independent, 1 if perfectly correlated, -1 if perfectly polar) times the product of the standard deviations:

\[ COV(x, y) = CORREL(x, y) \times SD(x) \times SD(y) \]
2-asset weighted portfolio variance

\[ V(aX + bY) = a^2V(X) + b^2V(Y) + 2ab\text{COV}(X,Y) \]

and in a weights sum to one, the coefficients above are restricted to the condition

\[ \sum \alpha_i = 1 \]

so in the special case of portfolio \( P \) consisting of two completely independent stocks with exactly the same variance and each equally represented, then the variance of the portfolio will be ...

\[ V(P) = 0.5V(X) \]

so if you only invested in one of the two stocks your volatility would be \( \sqrt{V(X)} \)

*but* if you diversified your portfolio 50/50 your volatility would be \( 0.707 \sqrt{V(x)} \)
Simple example of diversification using our formula:

Suppose you have two uncorrelated stocks, \( X(\mu, \sigma) \), \( X_1(0.02, 0.03) \) and \( X_2(0.04, 0.05) \). If you are risk-adverse, you may want to put all of your money in stock \( X_1 \) and accept the lower 2% yield. But what if you split your portfolio 50/50, giving you a 3% yield? What would your risk be??

\[
V_1 = 0.0009 \quad \text{and} \quad V_2 = 0.0025 \quad \text{and each alpha equals 1/2. Therefore}
\]

\[
V_{1,2} = 0.25 \times (0.0009 + 0.0025) = 0.00085
\]

\[
\sigma_{1,2} = 0.00085^{1/2} = 0.0291.
\]

Therefore, by diversifying your portfolio you have raised your yield to 0.03, 50% more than the conservative stock, while lowering your risk to a level \textit{below} the most conservative of the two stocks (which was at 0.03).
The risk-yield efficiency frontier

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Vol</th>
<th>Var</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>X2</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X1a</th>
<th>X2a</th>
<th>PVar</th>
<th>PVol</th>
<th>Palpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.0025</td>
<td>0.0500</td>
<td>0.0400</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>0.0020</td>
<td>0.0451</td>
<td>0.0380</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
<td>0.0016</td>
<td>0.0404</td>
<td>0.0360</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>0.0013</td>
<td>0.0361</td>
<td>0.0340</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>0.0010</td>
<td>0.0323</td>
<td>0.0320</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.0009</td>
<td>0.0292</td>
<td>0.0300</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>0.0007</td>
<td>0.0269</td>
<td>0.0280</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3</td>
<td>0.0007</td>
<td>0.0258</td>
<td>0.0260</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>0.0007</td>
<td>0.0260</td>
<td>0.0240</td>
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<tr>
<td>0.9</td>
<td>0.1</td>
<td>0.0008</td>
<td>0.0275</td>
<td>0.0220</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.0009</td>
<td>0.0300</td>
<td>0.0200</td>
</tr>
</tbody>
</table>
If we have 'n' assets in the portfolio, then we calculate the variance using this additive formula:

\[
VAR\left[\sum_{i=1}^{n} x_i\right] = \sum_{i=1}^{n} VAR(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=(i+1)}^{n} COV(x_i, x_j)
\]

which is easy to program if* you have the data. What finally matters, of course, is the square root of this term, the standard deviation, which is our volatility measure.

*this requires the calculation of all 'n' standard deviations and all paired correlations (15 for 6 stocks).

For reference and discussion, see http://mathworld.wolfram.com/Variance.html
Weight-adjusted n-asset Portfolio Volatility

If you assign weights to your portfolio, represented here as alphas, which of course you would, then the variance formula is:

\[
VAR\left[ \sum_{i=1}^{n} \alpha_i x_i \right] = \sum_{i=1}^{n} \alpha_i^2 VAR(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=(i+1)}^{n} \alpha_i \alpha_j COV(x_i, x_j)
\]

The volatility of this portfolio, the standard deviation, is the square root of this expression. Clearly, the greater the independence of your portfolio components, the smaller the risk. This shows the benefits of diversification into \textit{non-correlated} stocks.
Coding the Covariance (prior 2 equations)

For the covariance part of the equation only,

for I = 1 to (n-1) do
  for J = (i+1) to n do
    COV(I,J) = CORREL(I,J) * SD(I) * SD(J);
    SUMCOV = SUMCOV + COV(I,J);
  end;
end;

SUMCOV = 2*SUMCOV;

and the weighted portfolio calculation would be the same except

WCOV(I,J) = a(I)*a(J)*CORREL(I,J)*SD(I)*SD(J);

\[
\begin{bmatrix}
S_1 & S_2 & S_3 & S_4 \\
0 & C_{12} & C_{13} & C_{14} \\
0 & 0 & C_{23} & C_{24} \\
0 & 0 & 0 & C_{34} \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4
\end{bmatrix} \times 2
\]

Memo slide for sticklers for accuracy (a desirable trait), those of you who want to work in finance, and you coders who own a laptop and want to retire before age 35 trading off of any beach with a wireless setup.
Read the quotations from When Genius Failed ... 

... and lets do this in class together.