Futures Hedging and Contract Limit Models

Risk reduction with futures

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CMEGroup S&P 500 contract

Contract Specs:
1. 250 X S&P 500
2. $ or €
3. Symbol: SP
4. Current notional value: $450,000 (1800)
5. Initial margin: $23,788
6. Maint margin: $21,625
7. (also 50 X e-Mini), IM: $4,758!!

\[ \beta = \rho_{X} \frac{\sigma_{X}}{\sigma_{S}} \]

Source: CMEGroup

Specs on this provide the correlation coefficient and the denominator for our Beta formula on the left, and options on this contract provide the basis for calculating the VIX.
What we want to do here ...

- Remind ourselves of the limits of hedging.
- Cross-hedging and the **Minimum Variance Hedge Ratio**.
- Portfolio hedging with Index futures.

**Part 2**

- Explain how pure arbitrage works
- ... and how that puts **upper and lower price limits** on some futures
- The connection to **contango** and **backwardation**

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**Reasons to hedge – the example of copper**

**Contract Specs:**
1. 25,000 pounds
2. $ and cents per lb.
5. Initial margin: $7,763
6. Maint margin: $5,750
7. (also a 12,500 pound e-Mini)

Suppose you manufacture copper rotors induction motors, currently popular because they use no rare earth elements, and you use 23,000 tons of copper per year. You are in a competitive industry and a large price increase could bankrupt you if not hedged. You would have to be long about 150 contracts per month in the futures chain.
Is hedging always possible?

Of course not. If you need to hedge long and prices are at the top of their range, making it obvious that you should have hedged a year ago, it's too late. What good is it to hedge long on crude oil if the current spot and futures are above $120?

Also, if the contract is in contango we probably can’t hedge.

Rule of thumb: Probably the best time to hedge is when the current market makes it seem like you don't need to do it (i.e. when spot prices are very favorable).

From the perspective of most CFOs, the hedge buys certainty:

"You're basically buying a level of certainty," said John Heimlich, vice president and chief economist for the Air Transport Assn. "The market price may be higher, it may be lower, but I know what I'm going to pay, and I can set my business plan accordingly."

Airlines Again Consider Locking In Jet Fuel Prices, Los Angeles Times  September 5, 2006

The downside of hedging

We know that if, for example, you take a long hedge to avoid the cost of rising prices, if the price of the commodity declines after the hedge, then you will have to deal with the cash requirements of settlement, which can be considerable with a large hedge like jet fuel. This presents a potential interim cashflow problem even though the hedge is locking you into a known price.

Likewise not all executives understand the hedge all that well under these circumstances. As Hull points out, "It is easy to see why treasurers are reluctant to hedge. Hedging reduces risk for the company. However, it may increase risk for the treasurer if others do not fully understand what is being done." (page 52)
Cross hedging
(and cross speculating)

You may want to hedge, but there are a limited number of commodities traded with futures contract.

Suppose that you are an airline satisfied with the current spot price of Jet A-1 jet fuel and want to hedge (long) this price in the future. There is no futures contract for Jet A-1. But you know that jet fuel and RBOB Gasoline are likely to be strongly correlated. So maybe you can hedge a Jet A-1 contract by using a RBOB Gasoline futures contract.

But what if the volatility of Jet A-1 is much greater or lesser than RBOB Gasoline (even if the mean rates of price change are about the same)? This is going to make our hedge imperfect unless we adjust for it. (See example 3.3 in Hull).

Airlines Again Consider Locking In Jet Fuel Prices

*The Los Angeles Times, September 5, 2006*

"Airlines that pay for their jet fuel when they fill up their planes have been shelling out well over $2 a gallon lately — nearly four times the average price they were paying four years ago.

High fuel prices have dealt a much milder blow to carriers that have used a practice known as fuel hedging. It most often involves purchasing futures contracts that allow airlines to fix or cap the price they'll pay several months or years in advance. ....

In the last several years, Southwest has reaped sweeter rewards from fuel hedging than any other airline in the industry — nearly $1.8 billion in savings from 1999 to 2005.

Southwest had 85% of its fuel hedged at a rate based on $26 a barrel for crude last year, when oil was often trading at twice that price. That shaved about $892 million off the company's 2005 fuel bill.

Seven years ago, the Dallas-based low-fare carrier set a goal of having most of its projected fuel consumption hedged, said Laura Wright, the company's chief financial officer.

"We sleep better at night," she said, "if we know what that cost is going to be."

Note: This article included the suggestion that airlines can directly hedge jet fuel prices, which they cannot because there is not futures contract.
Cross-hedging: 
The minimum-variance hedge ratio (the Beta)

One of the most common uses of cross-hedging and related trades is to hedge a stock portfolio that you have constructed with a commonly-traded index futures contract, like the e-Mini S&P500 futures contract. Another application might be to hedge a fuel not represented in a futures contract with one that is, like jet fuel (JA-1) with crude oil or gasoline. In such an application, a minimum variance hedge ratio can be calculated (sometimes using spot and futures as below, or using both spot):

\[
\begin{align*}
\sigma_s &= \text{standard deviation of delta spot} \\
\sigma_f &= \text{standard deviation of delta future} \\
\rho &= \text{correlation coefficient of the two} \\
h &= \text{hedge ratio}
\end{align*}
\]

In the JetA-1 hedge, spot is of JetA-1, the future is of the RBOB Gasoline contract.

Cross-Hedging:  
Calculating the optimal number of contacts

Problem: How many contracts should you purchase when you are trying to hedge a position?

For example: Suppose you want to hedge (because you are buying) 1 million gallons (24 thousand barrels) of JA-1 in June. What sized long June contract should you buy?

Answer: Use this formula

\[
N = h \frac{S}{Q_f}
\]

where
N = optimal number of contracts
h = MV hedge ratio
S = size of position
Q_f = size of one contract
Estimation of MVHR by researchers

<table>
<thead>
<tr>
<th></th>
<th>1 year of historical data</th>
<th>2 years of historical data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regression Coefficient (H)</td>
<td>Correlation of Returns with Jet Fuel</td>
</tr>
<tr>
<td>Jet Fuel</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>1.06</td>
<td>77.00%</td>
</tr>
<tr>
<td>Heating Oil</td>
<td>1.15</td>
<td>90.35%</td>
</tr>
</tbody>
</table>

Correlation: 0.622584
SDJA-1 0.016241
SDGas 0.024107
Beta 0.419438
Weekly n=104

Source: Historical commodity prices from Datastream and U.S. Energy Information Administration

Note: This article points out that some airlines prefer to use options on futures rather than futures.

From

The connection between the MVHR, the Beta, and Correlation: The difficult side of cross hedging

The formulas for the MVHR and the Beta are the same, and frequent references to market “correlations” also refer to the same, so the application depends upon context.

Actually using the data introduces problems too. Do you use daily data, which reduces correlation, or weekly? Do you use CGRs (our approach) or deltas (Hull approach)?

And sometimes it just doesn’t come out the way you would think! Look at the Betas for JA-1 versus RBOB Gasoline and WTI, 2 years weekly data (found when setting up a HW for you):

<table>
<thead>
<tr>
<th></th>
<th>Correl: 0.622584</th>
<th>SDJA-1: 0.016241</th>
<th>SDFas: 0.024107</th>
<th>Beta: 0.419438</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily n=252</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jet Fuel</td>
<td>0.0273</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RBOB Gas</td>
<td>0.0393</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correl: 0.6530</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta: 0.4524</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly n=104</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jet Fuel</td>
<td>0.0273</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WTI Crude</td>
<td>0.0357</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correl: 0.8116</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta: 0.6204</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Weekly n=104
Hedging a stock portfolio with the S&P500 futures index

Suppose (for whatever reason) you wanted to completely hedge a stock portfolio. To do this, you would first have to calculate the Beta of the portfolio relative to the S&P 500 index (in the example below, think of a Beta like 1.5). Then, you would short an S&P500 futures contract using the formula below:

\[ N = \beta \frac{S}{F} \]

where
- \( N \) = number of S&P500 contracts
- \( \beta \) = the Beta
- \( S \) = the value of the portfolio
- \( F \) = the size of an S&P500 futures contract

The theoretical connection between pure arbitrage of storable commodities and upper and lower price limits ...
Pure (true) Arbitrage

Arbitrage takes advantage of the fact that two or more commodities or financial assets are mispriced relative to each other. With true arbitrage, an automatic profit will be realized.

*Usually arbitrage involves taking a long or short position in the primary asset and taking an opposite position in its derivative.*

Suppose the spot price of gold is $1200 per ounce. Suppose the one-year future price of gold is $1800 per ounce. The interest rate for borrowed money is 5% ($60 per ounce). The storage cost of gold is $40 per ounce. The insurance cost for gold is $20. The initial margin is 10% of the price of gold ($120) and the interest charge on that is $6.

What do you do?

1. Borrow $1200 and pay $60 interest.
2. Buy, store, and insure one ounce of gold.
4. Pay interest on the initial margin (implicitly)

Your “**carry cost**” on this contract is $126 = $60 + $40 + $20 + $6.

No matter what happens to the price of gold, you are guaranteed an arbitrage profit of $474 per ounce.

**Carry cost:** the cost of being in a futures contract, sometimes expressed as a percentage of the value of the contract. Consists of financing costs (direct or implied), plus storage, transportation, and insurance costs for relevant (storable) commodities.
Results from possible future gold prices

In one year you sell the gold that you bought for $1200 at the new spot price (5 possibilities are shown below). You add the gain or loss on the margin account (you were short at 400, so the margin account grows when the price of gold falls).

<table>
<thead>
<tr>
<th>Spot at end of year</th>
<th>Spot gain/loss</th>
<th>Margin gain/loss</th>
<th>less carry</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>-200</td>
<td>800</td>
<td>-126</td>
<td>474</td>
</tr>
<tr>
<td>1200</td>
<td>0</td>
<td>600</td>
<td>-126</td>
<td>474</td>
</tr>
<tr>
<td>1400</td>
<td>200</td>
<td>400</td>
<td>-126</td>
<td>474</td>
</tr>
<tr>
<td>1600</td>
<td>400</td>
<td>200</td>
<td>-126</td>
<td>474</td>
</tr>
<tr>
<td>1800</td>
<td>600</td>
<td>0</td>
<td>-126</td>
<td>474</td>
</tr>
<tr>
<td>2000</td>
<td>800</td>
<td>-200</td>
<td>-126</td>
<td>474</td>
</tr>
</tbody>
</table>

Future price upper limit (continuous)

Because the arbitrage possibility in the previous example would result in heavy purchases of gold at spot, raising the spot price, and heavy sales of gold futures, lowering the futures price, the spread in prices would narrow.

This implies that, given borrowing, storage, insurance, and other costs, there is an upper limit on futures prices for gold and similar commodities:

\[
FPUL = Se^{[r+s+i+r(m)t]t}
\]

Note: This limit exists for storable commodities only if storage is truly available!

FPUL = Future price upper limit
r = interest rate for borrowing
s = storage cost as a percent
i = insurance as a percent
m = initial margin
t = days in contract/365
S = Spot price

Discrete:

\[
FPUL = S[1 + r + s + i + r(m)]^t
\]
Arbitrage setting a lower limit

Suppose the spot price of gold is $1200 per ounce and the one-year futures price is $900.

1. Short one ounce of GLD
   • and pay the margin rate of 5% per $1200.
2. Go long in a futures contract for one ounce at $900
   • and pay a margin rate of 5% of the 10% initial margin.
3. Effectively take delivery of gold in the futures contract
   deliver it to satisfy your gold short position ...
   • by actually using the profit in you margin account to
     cover your short loss or the gain in your short position to
     cover your margin loss ... see example next slide

Results from possible future gold prices

<table>
<thead>
<tr>
<th>Spot at end of year</th>
<th>Short GLD gain/loss</th>
<th>Margin gain/loss</th>
<th>less carry</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>400</td>
<td>-100</td>
<td>-64.50</td>
<td>235.50</td>
</tr>
<tr>
<td>900</td>
<td>300</td>
<td>0</td>
<td>-64.50</td>
<td>235.50</td>
</tr>
<tr>
<td>1000</td>
<td>200</td>
<td>100</td>
<td>-64.50</td>
<td>235.50</td>
</tr>
<tr>
<td>1100</td>
<td>100</td>
<td>200</td>
<td>-64.50</td>
<td>235.50</td>
</tr>
<tr>
<td>1200</td>
<td>0</td>
<td>300</td>
<td>-64.50</td>
<td>235.50</td>
</tr>
<tr>
<td>1300</td>
<td>-100</td>
<td>400</td>
<td>-64.50</td>
<td>235.50</td>
</tr>
</tbody>
</table>

Note that the short position does not require storage nor insurance costs, at least not in this example. The articulation of variables depends upon what must actually be financed to take the position.
Arbitrage restricts futures price ranges

\[ FPUL = Se^{(r+s+i+r(m))t} \]

\[ FPLL = Se^{-[r+r(m)]t} \]

Note: This is true only if arbitrage is possible, which in the case of physical commodities, requires the means and possibility of storage.

The continuous case for 2012 Nat Gas

\[ FPUL = Se^{(r+s+i+r(m))t} \]

\[ FPLL = Se^{-[r+r(m)]t} \]

Contango region

Backwardation region

12% carry  8% short decay
Arbitrage with the S&P500 and other indexes

Arbitrage with the S&P500 index is the basis of what is called *Program Trading*, which is done with computers. Generally, this kind of arbitrage is done by buying a portfolio of the S&P500 stocks (or an index mutual fund) and at the same time shorting one of the S&P500 futures. This is a closed contract. Generally you are buying the stocks with the intention of delivering them (or the equivalent) when the futures contract expires. (This can also be done with SPD, the S&P ETF).

Why would you do this?

Class discussion.

### Upper/lower arbitrage ranges for the S&P500 futures contracts

<table>
<thead>
<tr>
<th>Months</th>
<th>10% plus</th>
<th>10% minus</th>
<th>6% plus</th>
<th>6% minus</th>
<th>3% plus</th>
<th>3% minus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep</td>
<td>1,606.63</td>
<td>1,554.91</td>
<td>1,531.42</td>
<td>1,509.00</td>
<td>1,486.57</td>
<td>1,464.05</td>
</tr>
<tr>
<td>Oct</td>
<td>1,591.80</td>
<td>1,539.22</td>
<td>1,516.64</td>
<td>1,494.05</td>
<td>1,471.45</td>
<td>1,448.84</td>
</tr>
<tr>
<td>Nov</td>
<td>1,535.01</td>
<td>1,482.42</td>
<td>1,459.84</td>
<td>1,437.25</td>
<td>1,414.65</td>
<td>1,392.05</td>
</tr>
</tbody>
</table>

**Region: Long on futures contract, short on spot. Borrow to go short.**

**Region: Long on spot, short on futures contract. Borrow to go long.**

- Sep 08 futures price at 1568.75
- Interest carry costs

1571.38 (+3% limit)
Definition of contango...

Most of the literature refers to any schedule of futures prices that is above the spot price and rising as the maturity extends, such as the graph on the left, as a contango. Because that is so common, that is the definition that we will use.

In some literature, a contango is defined as a pricing situation only where the futures prices is above the expected spot price, as shown in the bottom graph.

The latter definition gives rise to the possibility of a rising futures chain that is nonetheless not in contango.

This is relevant because in arbitrage we will consider the Contango Spread.

Contango theory: Why the futures price is not necessarily the expected future spot price

Assumes that

- In any market, hedgers are on one side net (say net long) and speculators are on the other (net short).
- Speculators as a group are able to estimate the future spot price of a commodity, and they might agree.
- Speculators will go short in a future only if the futures price is above the estimated future spot price of a commodity, otherwise for them it is a zero sum game with risk.
- Therefore, hedgers must be willing to buy long at a price above the estimated future spot price, or the market won’t clear – they are paying an insurance rate if they do this.
- Therefore, when the market clears, the futures price will be different than the expected future spot price.
Example of “true” contango where $FP > EFSP$

Suppose the price of wheat is currently $4.20 per bushel. Suppose that both millers and speculators think that the future spot price of wheat in six months will be $4.60.

Millers will want to “lock in” the price that is paid for their wheat by going long in the wheat futures market at $4.60, given that is still an acceptable value.

However, speculators will not agree to going short unless the price is above $4.60.

Therefore, if the millers want to hedge, they must agree to go long at a price above $4.60, or the market won’t clear, even though they estimate the future spot price to be $4.60. Therefore the market might clear at $4.80.

Contango/Backwardation possible scenarios

<table>
<thead>
<tr>
<th>Expected future spot price rising</th>
<th>Hedgers net long</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected future spot price falling</td>
<td>Hedgers net short</td>
</tr>
</tbody>
</table>

Red and Green lines show the actual futures prices relative to expected future spot prices (blue) depending upon whether hedgers are net long or net short in the futures contract in question.
Convergence of futures prices to spot prices expected by hedgers and speculators

(Hedgers net long, which implies speculators net short)

The green arrows show the effective insurance premium (spread) – note that this turns this into a positive-sum game for speculators.